



Dual-polarization telescope system based on lens doublets with a geometric-phase lens

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ABSTRACT

In this work we propose a dual-polarization telescope system based on two lens doublets, each composed of a geometric-phase (GP) lens and a conventional refractive lens. The bifocal properties of this hybrid lens doublet are obtained using the ray matrix formalism. First, we demonstrate a lens doublet design that exhibits two real accessible back focal planes with two different focal lengths (f_+ and f_-) for right- and left-handed circular polarizations (RCP and LCP). We then demonstrate the coupling of two such doublets to construct the dual polarization telescope, which yields two images with distinct magnifications for orthogonal circular polarizations. Here, the symmetry properties and RCP/LCP conversion of GP lenses are crucial in providing a magnified image for one circular polarization and a demagnified image for the opposite polarization. The conditions required for these two images to lie in a common plane are obtained. Experimental validation of the dual polarization telescope is demonstrated using two lens doublets, each combining a liquid-crystal GP lens and a standard refractive lens.

1. Introduction

Geometric-phase (GP) lenses are planar diffractive waveplate optical elements (DOEs) based on the Pancharatnam–Berry phase that exhibit polarization-dependent focusing properties [1]. These devices can be fabricated by direct ultrafast laser nano-structuring [2], with metamaterials (then known as metalenses) [3], or using liquid-crystals (LC) [4,5]. Metalenses exploit the artificial form-birefringence created by properly nanostructuring a material, and they can be designed with independent focal lengths for each circular polarization [6]. Aberration corrected doublets for compact imaging have been demonstrated by combining two such metasurface lenses [7,8]. Also, non-local metalenses capable of wavelength-selective directional focusing with a high forward-to-backward power ratio hold promise for LIDARs and optical gates [9,10].

On the contrary, liquid-crystal GP lenses exploit the natural birefringence of LC materials and the geometric phase is encoded in the in-plane patterning of the LC optical axis using photoalignment techniques

[11]. This enables practical advantages compared with GP metalenses. They can be produced in cost-effective facilities, with continuous spatial variation of the LC molecules, which avoids discrete steps –thus minimizing scattering and parasitic diffraction orders– and with large aperture areas [12]. Furthermore, they can be made electrically tunable by applying a voltage across the LC cell [13]. However, as opposed to metalenses, LC GP lenses are limited to provide the same focal length with opposite sign for right (RCP)- and for left (LCP)-handed circularly polarized beams. Despite this limitation, the study of optical systems based on liquid-crystal GP lenses remains highly relevant. Some efforts have focused on designing compound GP-lens systems that yield polarization-independent imaging [14]. Alternatively, their polarization-splitting capability has been exploited in applications such as polarization-sensitive zoom lens systems [15], vector beam analyzers [16], image-inversion interferometers for spatial-parity analyzers [17], polarization-sensitive Fourier transform processors [18] and compact achromatic Fourier transform systems [19].

In this work, we aim to realize polarization-sensitive lens systems

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based on LC GP lenses that provide different focal lengths for RCP- and LCP-handed circularly polarized beams without being restricted to the $\pm f$ condition intrinsic to standalone LC GP lenses. Our goal is to obtain two real back focal planes that are polarization-dependent and accessible. This is achieved by combining a GP lens with a conventional refractive lens whose focal properties are independent of the incident polarization state. We present a thorough analysis of the focusing properties of this lens doublet as a function of the focal lengths of both elements and their separation. In the framework of the first-order optics ABCD ray matrix formalism, we determine the locations of the focal and principal planes and the focal length of the system for each circular polarization component.

Based on these results, we design and implement a dual polarization-sensitive telescope whose magnification depends on the input state, providing magnification for one circular polarization and demagnification for the opposite circular polarization. The telescope is constructed using two GP-standard lens doublets, where the proper orientation of the GP lenses must be considered according to their symmetry and polarization-transforming properties [18]. We derive the conditions under which both polarization-dependent images are formed on the same output plane, enabling their superposition.

The paper is organized as follows: after this introduction section, Section 2 applies the ABCD ray-matrix formalism to analyze the bifocal properties of the GP – standard lens doublet, determining the location of the principal planes, the location of the focal planes, and the focal lengths for the RCP and LCP states as a function of the lens separation. In Section 3 these polarization-splitting properties are experimentally verified using a liquid-crystal GP lens and a conventional refractive lens. Then, Section 4 analyzes the conditions for building a dual polarization-sensitive telescope based on two such doublets, which provides magnified/demagnified images of the same object on a common image plane. The design leverages magnification ratio and telescope length. Experimental verification of the designed dual polarization telescope is included in Section 5. Finally, Section 6 presents the conclusions and perspectives of the work.

2. Bifocal properties of a lens doublet with a geometric-phase lens

GP lenses are diffractive lenses which exhibit two optical powers $\pm \mathcal{P}_G$, or equivalently two focal lengths $\pm f_G$, positive for input RCP and negative for input LCP or vice versa, depending on the lens orientation [20]. Thus, they are bifocal polarization-splitting lenses. Here, we consider the combination of a GP lens with a conventional lens to build a bifocal lens doublet, where the two focal lengths are not restricted to be the same with opposite sign.

Simple GP lenses, like those used here, exhibit a strong chromatic aberration characteristic of diffractive lenses [21]. This can be a very relevant problem in imaging systems. However, here, for the sake of simplicity, the analysis is limited to first-order optics and aberrations are neglected. Since the system is experimentally verified with a monochromatic laser, chromatic aberrations do not play a role. Note that non-local metalenses are wavelength-selective [9], thus if the system is built with such lenses the chromatic aberration drawback is minimized as well.

While the relations for the lens doublet are very well-known, here we apply the ABCD matrix method [22] for their derivation. This mathematical framework enables the analysis of ray propagation through optical systems and is fully compatible with the diffractive nature of GP lenses [23]. These matrices operate on paraxial rays $\mathbf{r} = (y, \theta)$, where y is the transverse ray height and θ is the paraxial angle. Its basic elements are the free-space propagation matrix ($\mathbf{D}$) and the thin lens matrix ($\mathbf{L}$), given respectively by:

$$\mathbf{D}(d) = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}, \quad \mathbf{L}(f) = \begin{bmatrix} 1 & 0 \\ -\mathcal{P} & 1 \end{bmatrix}, \quad (1)$$

where d and $\mathcal{P} = 1/f$ denote, respectively, the propagation distance and the lens optical power, with f being its focal length.

We consider a tandem configuration where a standard lens (S) is placed first, followed by the GP lens (G), with a separation distance d between them (Fig. 1). Such a doublet system is described by the matrix $\mathbf{M}_D$ that results from the following matrix multiplication:

$$\mathbf{M}_D = \mathbf{L}_G \times \mathbf{D} \times \mathbf{L}_S = \begin{bmatrix} 1 & 0 \\ \mp \mathcal{P}_G & 1 \end{bmatrix} \times \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ -\mathcal{P}_S & 1 \end{bmatrix}. \quad (2)$$

Note that in the $\mathbf{L}_G$ matrix above, $\mathcal{P}_G$ (and consequently f_G) is defined as an absolute value, with the $\pm$ sign explicitly accounting for the two possible values corresponding to the circular polarization states. Equation (2) can be written as:

$$\mathbf{M}_D = \begin{bmatrix} A_D & B_D \\ C_D & D_D \end{bmatrix} = \begin{bmatrix} 1 - \mathcal{P}_S d & d \\ \mp \mathcal{P}_G - \mathcal{P}_S \pm \mathcal{P}_S \mathcal{P}_G d & 1 \mp \mathcal{P}_G d \end{bmatrix}. \quad (3)$$

The ABCD elements of this ray matrix determine the first-order parameters of the optical system. For instance, the doublet optical power ($\mathcal{P}$) and focal length (f) are directly derived from parameter $C_D = -\mathcal{P} = -1/f$. Since the GP lens has two focal lengths $\pm f_G$, depending on the input polarization, two values are obtained for the doublet optical power:

$$\mathcal{P}_{\pm} = \mathcal{P}_S \pm \mathcal{P}_G \mp \mathcal{P}_S \mathcal{P}_G d, \quad (4)$$

and for the doublet focal length:

$$f_{\pm} = \frac{\pm f_G f_S}{f_S \pm f_G - d} \quad (5)$$

where subindices $+/-$ correspond to the positive/negative values of the GP optical power.

Since the system is a compound lens, it is necessary to determine the locations of the focal and principal planes. To find the focal planes, we add free-space propagations $\mathbf{z}_F = \overline{SF}$ in front of the first lens (S) and $\mathbf{z}_F = \overline{GF}$ behind the second lens (G). Note that the distance $\mathbf{z}_F$ has its origin in the first lens of the doublet and we follow the sign criteria for optical lens systems in Ref. [24]. Thus, the corresponding ray matrix (Fig. 1(b)) must account for a negative distance. The overall ray matrix is:

$$\mathbf{D}(\mathbf{z}_F) \times \mathbf{M}_D \times \mathbf{D}(-\mathbf{z}_F) = \begin{bmatrix} A_D + C_D \mathbf{z}_F & -A_D \mathbf{z}_F - C_D \mathbf{z}_F \mathbf{z}_F + B_D + D_D \mathbf{z}_F \\ C_D & D_D - C_D \mathbf{z}_F \end{bmatrix}. \quad (6)$$

To determine the back focal plane F of the lens doublet, the top left element of the ray matrix is set to zero [25], i.e. $\mathbf{z}_F' = -A_D/C_D$, which yields two different back focal planes (F_- and F_+) separated from the second lens (G) by the axial distances:

$$\mathbf{z}_{F_{\pm}}' = \frac{\mathcal{P}_S d - 1}{\pm \mathcal{P}_S \mathcal{P}_G d \mp \mathcal{P}_G - \mathcal{P}_S}. \quad (7)$$

Once the back focal planes $F_{\pm}$ are known, the corresponding back principal planes $H_{\pm}$ can be located at a distance $f_{\pm}$ from $F_{\pm}$. Using $f = \overline{HF}$, $\mathbf{z}_F = \overline{GF}$, and $\mathbf{z}_H = \overline{GH} = \overline{GF} + \overline{FH} = \mathbf{z}_F - f$ yields the two following locations of the principal planes, measured from the GP lens:

$$\mathbf{z}_{H_{\pm}} = \frac{\mathcal{P}_S d}{\pm \mathcal{P}_S \mathcal{P}_G d \mp \mathcal{P}_G - \mathcal{P}_S}. \quad (8)$$

Similarly, to determine the doublet front focal plane F , the right bottom element of the ray matrix must be null, i.e. $\mathbf{z}_F = D_D/C_D$, giving the two following distances

$$\mathbf{z}_{F_{\pm}} = \frac{1 \mp \mathcal{P}_G d}{\pm \mathcal{P}_S \mathcal{P}_G d \mp \mathcal{P}_G - \mathcal{P}_S}, \quad (9)$$

and the front principal planes $H_{\pm}$ are obtained using $\mathbf{z}_H = \overline{SH}$, $f = \overline{FH}$ and $\mathbf{z}_H = \overline{SH} = \overline{SF} + \overline{FH} = \mathbf{z}_F + f$, thus yielding the two following lo-

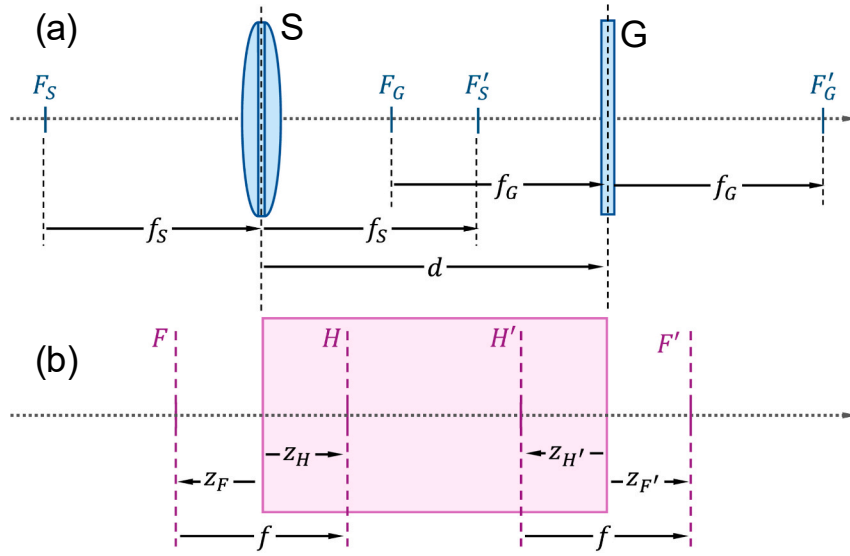


Fig. 1. (a) Schematic of a standard (S) lens - geometric-phase (GP) lens doublet. The focal length of the GP lens and the standard lens are, respectively, (f_G, f_S) . (F_G, F'_G) and (F_S, F'_S) indicate the front and back focal planes of the GP lens and standard lens respectively. (b) Description of the S-GP lens doublet in terms of its cardinal planes. (F, F') and (H, H') indicate the front/back focal and principal planes of the doublet respectively. $(z_F, z_{F'})$ indicate the distances from the S lens to the front focal plane F and from the GP lens (G) to the back focal plane F' respectively. $(z_H, z_{H'})$ indicate the distances from the S lens to the front principal plane H and from the GP lens (G) to the back principal plane H' respectively.

cations, measured from the refractive lens:

$$z_{H_{\pm}} = \frac{\mp \mathcal{P}_G d}{\pm \mathcal{P}_S \mathcal{P}_G d \mp \mathcal{P}_G - \mathcal{P}_S}. \quad (10)$$

Our aim is to design a convergent lens doublet where both polarization components focus on two real back focal planes (i.e. we want both f_- and f_+ to be positive). This way, the two back focal planes are accessible for potential filtering operations. If the two lenses are in contact ($d=0$) this implies that the power of the standard refractive lens must be larger than that of the GP lens. In this work, in contrast, we analyze the separated lens doublet ($d>0$), to discuss a more flexible situation.

As an example, Fig. 2 presents the results for a doublet matching the values in our experimental system. The GP lens has focal lengths $\pm f_G = \pm 91$ mm and the standard lens is selected with focal length $f_S = +100$ mm. The figure shows the theoretical location of the back focal and principal planes as a function of the lens separation d , indicated by distances z_F and z_H (Fig. 2(a)), and the front focal and principal planes, indicated by distances $z_{F'}$ and $z_{H'}$ (Fig. 2(b)). Fig. 2(c) shows the evolution of the doublet focal length f versus d for both polarization components. The short focal length of the doublet (f_+) is obtained when the GP lens acts as a convergent lens, while its long focal length (f_-) corresponds to the GP lens acting as a divergent lens. In our setup, this occurs for the RCP and LCP states, respectively. Note that the evolution with d is much stronger for the LCP state. This is a consequence of having selected $f_S \cong f_G$, which makes almost neglectable the effect of the GP lens on the converging action of the doublet.

Let us remark that, considering the distances $z_{F'_{\pm}}$ and $z_{F_{\pm}}$ in Eqs. (7) and (9), the matrix in Eq. (6) describing the light propagation from the front focal plane ($F_{\pm}$) to the corresponding back focal plane ($F'_{\pm}$) takes the following expression:

$$\mathbf{D}(z_{F'_{\pm}}) \times \mathbf{M}_D \times \mathbf{D}(-z_{F_{\pm}}) = \begin{bmatrix} 0 & f_{\pm} \\ -1/f_{\pm} & 0 \end{bmatrix}, \quad (11)$$

which corresponds to the ray matrix describing a Fourier transform optical system [22]. This relation will be used in Section 4 to analyze the dual polarization telescope.

Finally, note that the reversed doublet, where the GP lens is placed as the first lens, is described by a ray-transfer matrix like Eq. (3) but exchanging the diagonal elements. Hence, the reversed doublet has the same optical power and focal lengths as discussed in Eqs. (4) and (5). However, from symmetry relations, the locations of its focal and principal planes are reversed to the ones previously discussed, i.e.:

$$z_{F'_{\pm}}^{\text{rev}} = -z_{F_{\pm}}, \quad z_{H'_{\pm}}^{\text{rev}} = -z_{H_{\pm}}, \quad (12a)$$

$$z_{F_{\pm}}^{\text{rev}} = -z_{F'_{\pm}}, \quad z_{H_{\pm}}^{\text{rev}} = -z_{H'_{\pm}}. \quad (12b)$$

These relations will be used in Section 4 to build the dual polarization telescope.

3. Experimental validation of the dual polarization lens doublet

Fig. 3 illustrates the experimental setup to characterize the lens doublet prior to further applications. A collimated He-Ne laser beam ($\lambda = 632.8$ nm) serves as the light source. The polarization state entering the system is controlled with a polarization state generator (PSG) composed by a linear polarizer (P) and a quarter-wave plate (QWP). Before the PSG, an iris diaphragm and another polarizer were placed to control the area and the intensity of the input beam, respectively. The two input circular polarizations states are generated by rotating the QWP such that its axes are at $\pm 45^\circ$ relative to polarizer P, while linearly polarized states can be obtained when orienting the QWP at 0° or 90° . Note these changes in the polarization of the input beam are produced without change of its intensity. The lens doublet is built with a standard refractive lens with $f_S = +100$ mm and a liquid-crystal GP flat lens (Edmund Optics ref. #34-466, with nominal $f_G = \pm 100$ mm for green light). First, the actual focal lengths of the individual lenses were verified, yielding a shorter value of $f_G = \pm 91$ mm at the operating red wavelength, as expected in a diffractive lens. This agrees with the linear relation of $\lambda f_G = 58.87$ [nm·m] between the optical power $1/f_G$ versus wavelength obtained in Ref. [26]. Second, the wavelength-dependent retardance and polarization-conversion efficiency of the GP lens were measured following the spectral characterization procedure reported in Ref. [20]. This implied placing the GP lens between achromatic circular polarizers and using a supercontinuum laser (Fianium SC400) as

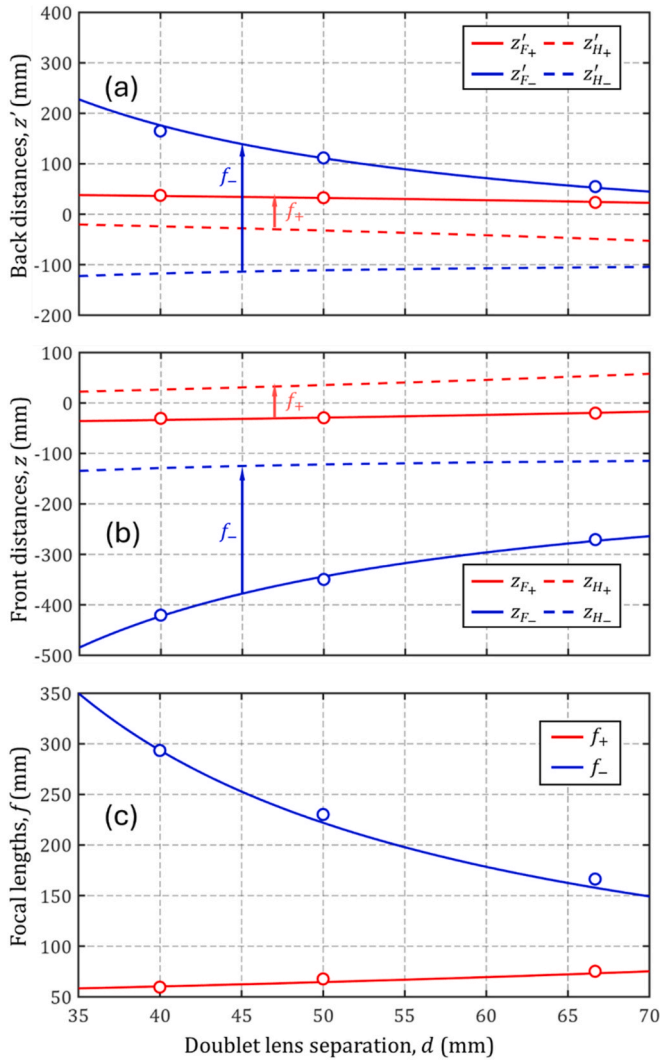


Fig. 2. Locations of the back planes (a), front planes (b), and focal lengths (c) of the lens doublet with $f_S = +100$ mm and $\pm f_G = \pm 91$ mm as a function of the doublet lens separation d , for the short focal length (red curves, convergent GP lens) and for the long focal length (blue curves, divergent GP lens). Circle symbols correspond to experimental measurements. (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)

broadband light source. The GP lens polarization-conversion efficiency η corresponds to the normalized transmittance between crossed circular polarizers: $\eta = \sin^2(\phi/2)$, where ϕ is the retardance of the LC layer,

given by: $\phi = (2\pi/\lambda)\Delta n d$, where λ is the wavelength, Δn is the LC birefringence, and d is the layer thickness. A polarization-conversion efficiency of 96% and 0.9π retardance at 632.8 nm was obtained for the GP lens, with a transmittance over 93% within the spectral range of 500–850 nm. Nevertheless, the good performance of this GP lens in terms of polarization conversion (efficiency of circa 95%) is only ensured in a narrower spectral range of 600–750 nm. Note that non 100% efficiency will result in non-focused parasite light.

The purpose of the experiment is to determine the positions of the focal planes and the focal lengths for each input polarization state, as the lens separation d is varied. Three values were chosen (40 mm, 50 mm, and 66.7 mm). For each separation d , the camera detector was displaced along the optical axis to find the two focal spots, corresponding to each input circular polarization. Here, no object was placed in front of the doublet, just the collimated polarized input beam. The measured values of z'_F are indicated in Fig. 2(a) as circles. The short focal plane position (z'_{F_+}) was obtained for input RCP while the long focal plane position (z'_{F_-}) was obtained for input LCP. The experimental positions of the focal planes show excellent agreement with theoretical predictions, with a maximum error of 4%. The experimental distances indicated in Fig. 2(b) for the front focal plane position (z_{F_+}) for input RCP and for input LCP (z_{F_-}) were obtained by detecting the back focal plane of the reversed doublet and then applying Eq. (12a).

Now, to experimentally determine the focal lengths f_- and f_+ of the lens doublet, we apply the scaling property of the optical Fourier transform (FT) obtained at the back focal plane of a lens. A diffraction grating is placed before the lens doublet, illuminated with a collimated beam. The diffraction orders are then focused on the back focal plane, and the scale is proportional to λf [27]. The quadratic phase term that arises if the grating is not exactly placed at the front focal plane is not relevant to this analysis, since we are capturing intensity.

Fig. 4 shows the FT captures at the focal planes F'_+ and F'_- for the doublet with the three considered separations ($d = 40, 50$ and 66.7 mm, respectively) and for input circular polarization states RCP and LCP. For each separation, the camera was displaced axially to the corresponding focal planes F'_+ and F'_- . Focused diffraction orders are evident in the F'_+ plane for input RCP, and in the F'_- plane for input LCP, but the separation between orders is much smaller in F'_+ . Defocused diffraction orders in plane F'_- interfere since they overlap.

From these images, the focal length can be retrieved by measuring the separation (Δx) between the $\pm$ first diffraction orders at each plane (Fig. 4). Forming the FT with a reference lens of known focal length (f_{ref}), yielding a separation Δx_{ref} between diffraction orders we can retrieve the individual focal length values f_- and f_+ as:

$$f_{\pm} = f_{ref} \frac{\Delta x_{\pm}}{\Delta x_{ref}}, \quad (13)$$

The single GP lens was selected as the reference lens since, being an

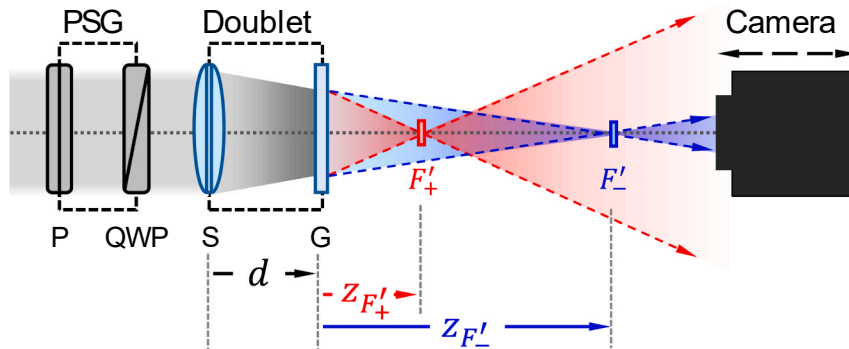


Fig. 3. Experimental setup to characterize a standard (S) refractive lens–geometric-phase (G) lens doublet. The polarization state generator (PSG) comprises a linear polarizer (P) and a quarter-wave plate (QWP).

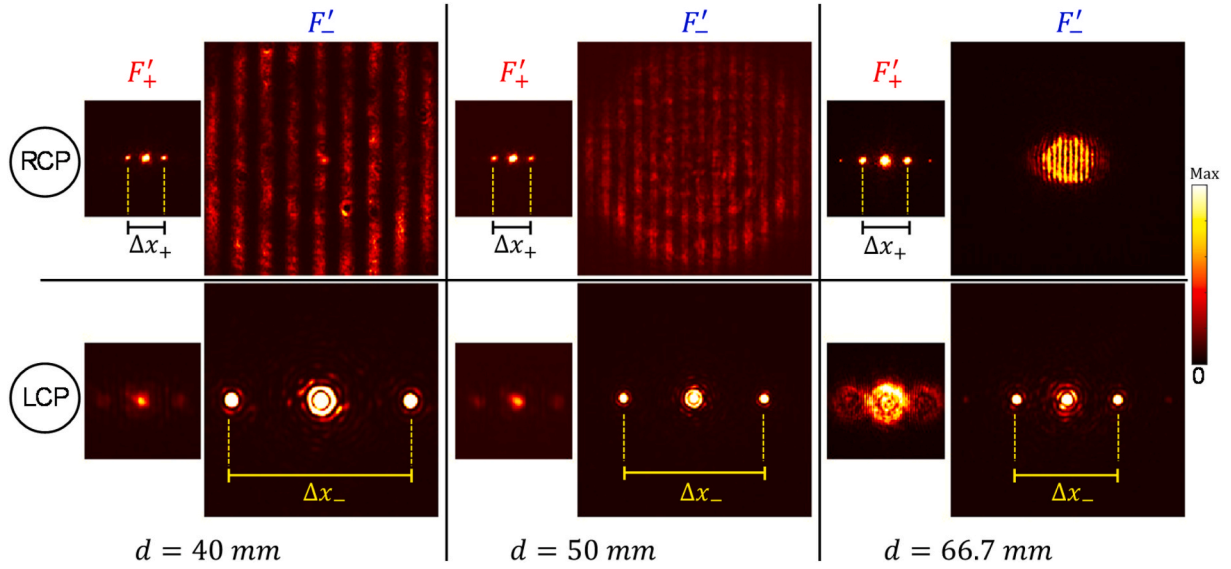


Fig. 4. Captures taken at the back focal planes of the test lens doublet of the Fourier transform of a diffraction grating, for three separation values d between lenses. The captures on plane F'_+ are reported on the left, while those on plane F'_- are reported on the right of each case. The top and bottom rows correspond to an input RCP/LCP beam. The separation $\Delta x_{\pm}$ between diffraction orders allows retrieving the focal lengths of the doublet through Eq. (13).

ultra-thin lens, the determination of its focal length is very precise. The experimental values of f_- and f_+ thus obtained from the FT captures for the three doublets considered are plotted in Fig. 2(c) as circles. They are again in very good agreement with theory.

4. Analysis and imaging properties of a dual polarization telescope

Once the bifocal polarization properties of the doublet have been verified, we are interested in using them to build a telescope that forms a coplanar image of a near object with different magnifications for

orthogonal polarizations. The goal is to achieve an optical system where the magnification can be switched by changing the input polarization. It is very well known that a telescope forms images of close objects with constant magnification, which depends only on the objective/eyepiece focal lengths ratio. Thus, the idea is to build a telescope where the objective lens and the eyepiece lens are both a lens doublet as described in the previous section. By properly coupling these two doublets, the telescope condition is fulfilled for both input circular polarization components, but the objective/eyepiece focal lengths ratio is inverted for one input circular polarization compared to the other. As a result, the telescopic system shows magnification/demagnification depending on

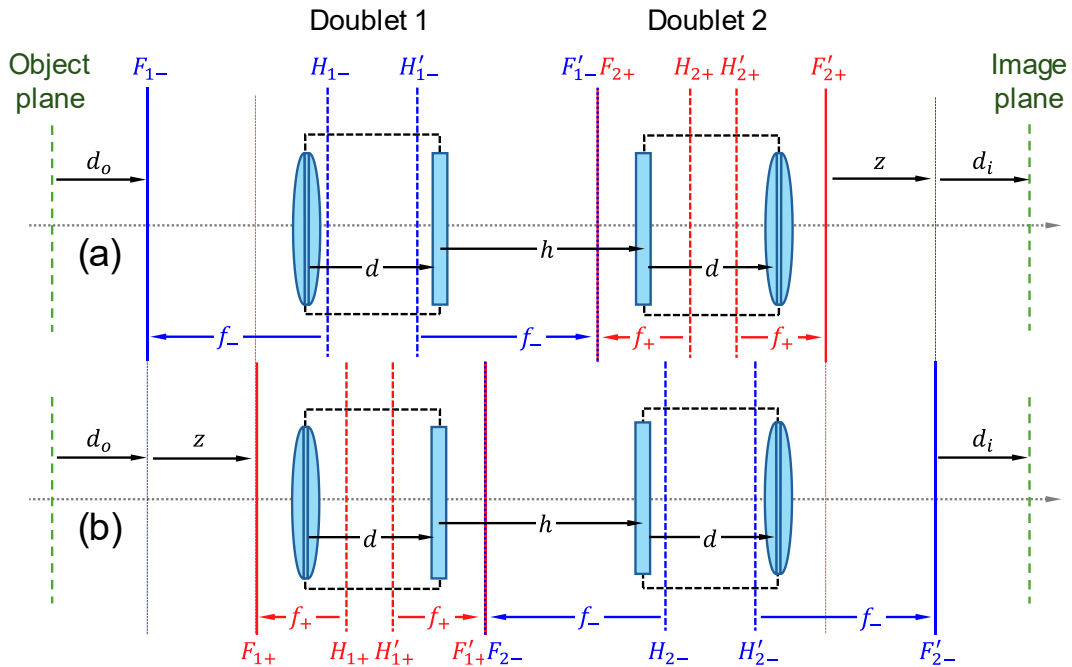


Fig. 5. Schematic of the dual polarization telescope as a tandem of two GP-standard-lens doublets. Location of the doublets' cardinal planes for (a) input LCP and (b) input RCP. $(F_{j\pm}, F'_{j\pm})$ and $(H_{j\pm}, H'_{j\pm})$ indicate the front/back focal and principal planes of each doublet ($j=1,2$) and subindices $\pm$ apply to RCP/LCP input polarization.

the input polarization. This idea was exploited in the past to create a zoom lens system [15] but using doublet lenses in contact ($d = 0$).

Fig. 5 represents our dual polarization telescope system based on two identical lens doublets with the same lens separation d . We select $d = 50$ mm, thus having focal lengths $f_+ = 64.5$ mm and $f_- = 221.9$ mm. The first doublet, doublet 1, is set by a refractive lens S1 and a GP lens G1. Doublet 2 is identical but placed with the reversed arrangement i.e. the GP lens G2 is in front of the refractive lens S2. The distance h between doublets is the distance from lens G1 to lens G2. To build the dual polarization telescope, the distance h must be such that, for each circular polarization, the back focal plane of doublet 1 coincides with the front focal plane of doublet 2. Due to the interchange of the RCP/LCP components after traversing the GP lenses, if G1 and G2 are placed with the same orientation, the input polarization that experienced G1 as a converging lens, now encounters G2 as a diverging lens, and vice versa. Therefore, using two identical doublets, with doublet 2 reversed with respect to doublet 1, the telescope condition is fulfilled simultaneously for the two polarization components if the doublets are separated a distance $h = z_{F_-} + z_{F_+}$.

Fig. 5(a) and (b) represent the corresponding telescope for input LCP and RCP components respectively. The telescope system, analyzed from the front focal plane of the first lens until the back focal plane of the second lens, can be viewed as a $4f$ system composed by two Fourier transform systems as in Eq. (11), i.e.:

$$\mathbf{M}_{T\pm} = \begin{bmatrix} 0 & f_{\mp} \\ -1/f_{\mp} & 0 \end{bmatrix} \times \begin{bmatrix} 0 & f_{\pm} \\ -1/f_{\pm} & 0 \end{bmatrix} = \begin{bmatrix} -m^{\pm 1} & 0 \\ 0 & -1/m^{\pm 1} \end{bmatrix}, \quad (14)$$

where subindices $\pm$ apply to input RCP/LCP, and where $m \equiv f_-/f_+ > 1$. Therefore, one circular polarization component is magnified by $-m$ while the orthogonal component is demagnified by $-1/m$.

Here we are interested in using the telescope for imaging a near object. To assess its performance, we add propagations on both sides of the telescope as shown in Fig. 5. For input LCP (Fig. 5(a)) the following ray matrix is derived:

$$\mathbf{M}_{\text{IM}(LCP)} = \begin{bmatrix} 1 & z+d_i \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} -\frac{1}{m} & 0 \\ 0 & -m \end{bmatrix} \times \begin{bmatrix} 1 & d_o \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{m} & -\frac{d_o}{m} - m(z+d_i) \\ 0 & -m \end{bmatrix}. \quad (15)$$

where d_o and $z+d_i$ are the object and image positions measured, respectively, from the object to the front focal plane of the first doublet (F_{1-}), and from the back focal plane of the second doublet (F_{2+}) to the image plane. The distance z is the distance between the two front focal planes of doublet 1, F_{1-} and F_{1+} , or, equivalently, between the two back focal planes of doublet 2, F_{2-} and F_{2+} . Thus, $z = F_{1-}F_{1+} = F_{1-}S_1 + S_1F_{1+} = -z_{F_{1-}} + z_{F_{1+}}$ which can be calculated from Eq. (9).

The imaging condition is obtained when $B = 0$ [22] in the matrix in Eq. (15), leading to the image distance:

$$d_{i(LCP)} = -z - \frac{d_o}{m^2}. \quad (16)$$

For input RCP, the same light propagation from object to image plane is indicated in Fig. 5(b). The resulting ray matrix is now given by:

$$\mathbf{M}_{\text{IM}(RCP)} = \begin{bmatrix} 1 & d_i \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} -m & 0 \\ 0 & -\frac{1}{m} \end{bmatrix} \times \begin{bmatrix} 1 & d_o+z \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -m & -m(d_o+z) - \frac{d_i}{m} \\ 0 & -\frac{1}{m} \end{bmatrix}, \quad (17)$$

and the imaging condition ($B = 0$) leads to

$$d_{i(RCP)} = -m^2(z+d_o). \quad (18)$$

Thus, the telescope provides two images, one for input LCP at

distance $d_{i(LCP)}$ with lateral magnification $-1/m$, and another for input RCP at distance $d_{i(RCP)}$ with magnification $-m$. In general, these two images appear at different planes, $d_{i(LCP)} \neq d_{i(RCP)}$. However, there is an object distance d_o where the two images appear at the same plane d_i . By setting Eqs. (16) and (18) equal to each other, this object distance d_o is given by:

$$d_o = -z \frac{m^2}{(1+m^2)} = (z_{F_{1-}} - z_{F_{1+}}) \frac{m^2}{(1+m^2)}. \quad (19)$$

In this situation (using Eq. (19) in either Eq. (16) or (18)), the symmetric condition for imaging is satisfied: $d_o = d_i$.

Our aim is to find the lens separation in the doublets (d) that allows for a considerable magnification ratio while seeking a compact telescope size. The telescope total length, from input lens to output lens, is $2d + h$, where $h = z_{F_-} + |z_{F_+}|$ and is a function of d according to Eqs. (7) and (9). Since the input RCP component is magnified by $-m$ while the LCP component is demagnified by $-1/m$, the magnification ratio between the two images provided by the telescope is m^2 . Fig. 6 shows m^2 and the telescope length $2d + h$ as the doublets' lens separation d varies within the same range as in Fig. 2, from 35 to 70 mm. The circle symbols refer to the three doublets considered in the previous section ($d = 40, 50$ and 66.7 mm). Larger d results in a more compact telescope, but at the expense of a smaller magnification ratio, i.e. the size of the magnified and demagnified images is similar. Decreasing the lens separation in the doublets leads to the opposite trend, i.e. a longer telescope and larger size difference in the final images.

5. Experimental validation of the dual polarization telescope

Fig. 7 shows the experimental setup of the dual polarization telescope. To validate the telescope imaging performance, a test object was illuminated with a collimated He-Ne laser beam ($\lambda = 632.8$ nm) whose polarization was controlled with the PSG. Once again, as in Section 3, before the PSG, an iris diaphragm and another polarizer (P1) were placed to control the area and the intensity of the input beam. The input beam illuminates the object plane where two objects that do not alter the

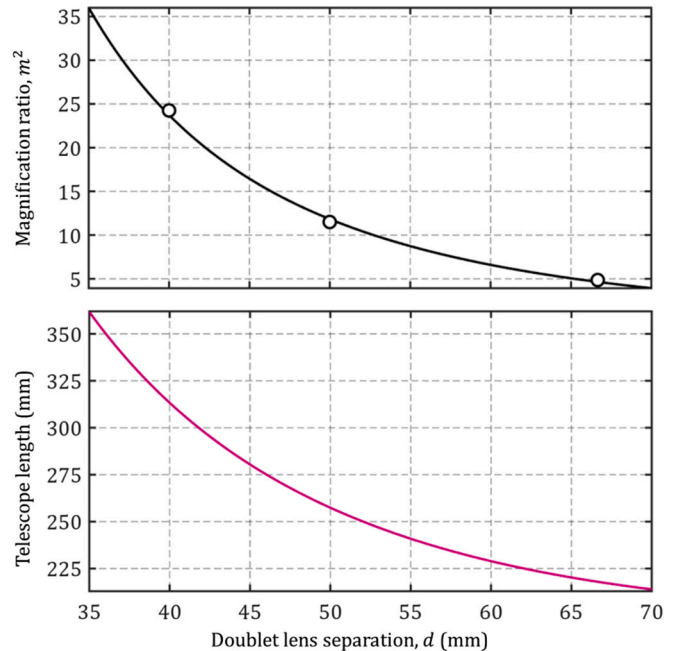


Fig. 6. Magnification ratio m^2 between the two images provided by the dual polarization telescope (solid line theory, symbols experiment) and telescope length $2d + h$ as a function of the doublet lens separation d .

polarization were employed: an adjustable rectangular slit and a resolution test (NBS 1963A, $2'' \times 2''$).

We built the dual telescope using two identical bifocal doublets as described in Section 2. Considering the physical limitations of the laboratory, a separation $d = 50$ mm was selected, for which the expected magnification ratio is $m^2 = 11.8$ and the telescope length is 257.4 mm. Since the experimental accuracy in our setup was ± 1 mm, the implemented h value was 143 mm. Note that, since each GP lens transforms the incident circular polarization into its orthogonal state, input RCP/LCP beams focus respectively on intermediate planes F'_+ and F'_- , and become LCP/RCP polarizations. They are then transformed back into RCP/LCP states by the second GP lens (G2), thereby recovering the initial state.

For practical purposes, we measure the object and image positions relative to lenses S1 and S2, respectively, as shown in Fig. 7. Therefore, considering the schematic in Fig. 5 and the focal plane distances, $-s = d_o - z_{F'_-}$ is the object distance and $s' = d_i + z_{F'_+}$ is the image distance in the laboratory frame. As discussed in the previous section, $d_o = d_i$. This symmetric imaging condition is maintained in the reference frame adopted in the experiment, i.e. $-s = s'$ because of the symmetry relation in Eq. (12a) by which the locations of the back and front focal planes of the reversed and standard doublet are reversed ($z_{F'_-} = -z_{F'_+}$). We can rewrite equation Eq. (19) to obtain the condition for the formation of two images on the same plane as a function of the telescope parameters. The image distance in the laboratory frame is then:

$$s' = \left(\frac{f_+}{1+m^2} \right) \left[\left(1 + \frac{d}{f_g} \right) m + \left(1 - \frac{d}{f_g} \right) m^2 \right]. \quad (20)$$

For the conditions of the experiment with $d = 50$ mm, the theoretical values for the object and image plane positions are $-s = s' = 53.6$ mm. These values were adjusted experimentally until an optimal imaging condition was achieved.

Fig. 8(a) shows the experimental results with the rectangular aperture as the object. By rotating the QWP in the PSG, the input polarization is changed from RCP to a linear state and to LCP. The image plane displays a magnified image for input RCP (Fig. 8(a)) and a demagnified image for input LCP (Fig. 8(b)). When aligning the QWP with the input polarizer, the input beam is linearly polarized and both images superimpose in the image plane (Fig. 8(c)). Since linear polarization has equal

weights of the circular polarization components, the demagnified image appears more intense than the magnified image, by a factor m^4 assuming the system exhibits the equivalent ray clipping for the two images. By measuring the size of the two images of the rectangular aperture it is possible to evaluate the magnification ratio, leading to an experimental value $m^2 = 11.9$.

The same procedure was performed with the resolution test as the object. Fig. 8(d) and (e) show the image plane for input RCP and LCP, respectively. Again, one magnified and one demagnified image of the input object are obtained. Finally, in Fig. 8(f) the orientation of the QWP was adjusted to provide an elliptically polarized input beam (with an ellipticity of 0.84), resulting in two images with the same level of intensity. In this case a magnification ratio $m^2 \approx 11.3$ was measured, again in good agreement with the theory.

As mentioned, the experimental accuracy in our setup is ± 1 mm. Therefore, a pertinent question arises on how sensitive the telescope is to deviations on the design parameters: focal length of the GP lenses Δf , doublet size Δd and separation between doublets Δh . The impact of these deviations on the telescope performance is assessed by calculating the relative error on the magnification ratio m^2 and on the image position s' for each circular polarization state. An error of $\Delta f = \pm 1$ mm in retrieving the GP-lens focal length yields a 3.4% variation of the magnification ratio and relative displacements of the LCP and RCP images of 18% and 1.6%, respectively. The length d of the lens doublet plays a central role because it determines the optical power of a single doublet while also fixing the telescope size and the symmetry/telescope condition. Relative errors in the image-plane location (driven by $\Delta d = \pm 1$ mm) are 1% for RCP and 11.4% for LCP, with a magnification variation of 6.1% with respect to the nominal value $d = 50$ mm. The deviation of $\Delta h = \pm 1$ mm in the separation between the doublets drives lower errors in the image-plane location s' : 0.3% and 3.2% for the RCP and LCP images, respectively. In this case, the magnification ratio relative error is 1.2%.

GP lenses present the chromatic variation of the focal length characteristic of diffractive lenses; namely, a linear relation between the optical power and wavelength [26]. Thus, we have also examined how adaptable the telescope architecture is to input wavelengths other than 632.8 nm. We considered the spectral range from 600 nm to 750 nm, where around 95% polarization-conversion efficiency is guaranteed for

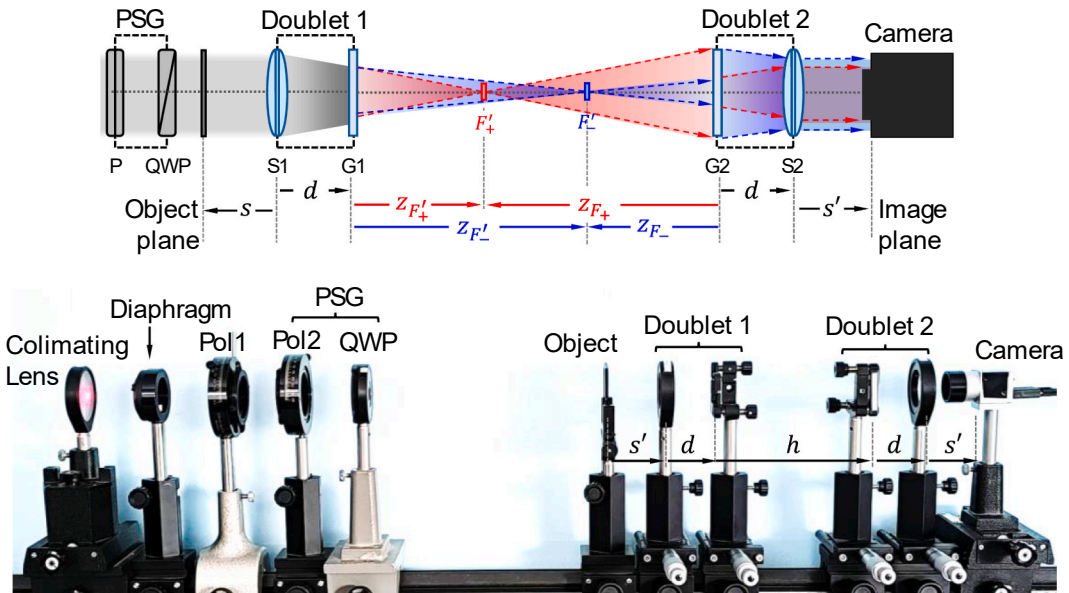


Fig. 7. Scheme of the polarization telescope for dual imaging a near object in a common image plane (top) and picture of the experimental system (bottom). The linear polarizer (Pol1) controls the intensity of the input beam, and the diaphragm controls the beam size. The polarization state generator (PSG) comprises a linear polarizer (P) and a quarter-wave plate (QWP). The system includes two standard (S) refractive lenses and two geometric-phase (G) lenses.

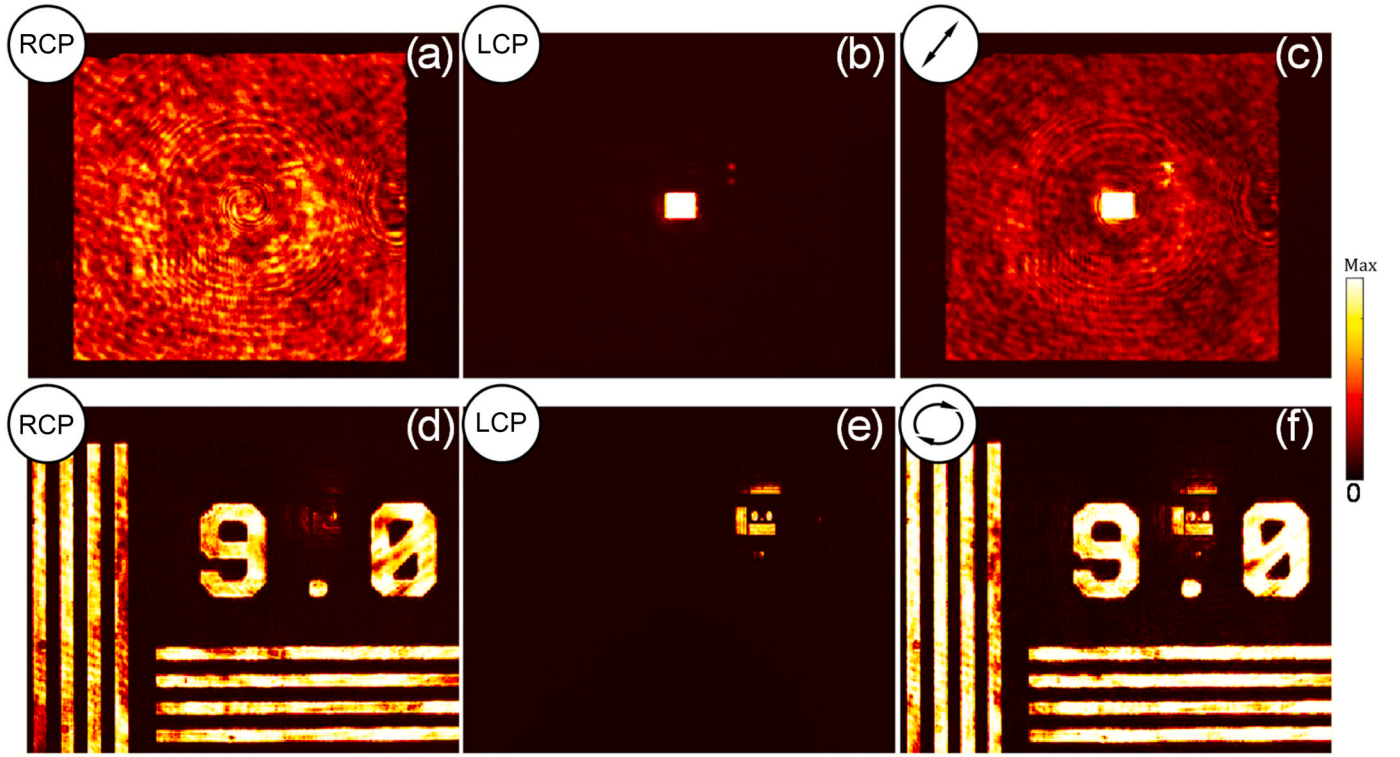


Fig. 8. Captures showing the imaging performance of the dual polarization telescope with $d = 50$ mm and $h = 157.4$ mm for different input polarizations, using a rectangular slit as an object (top row) and using a test object (bottom row). (a,d) input RCP, (b,e) input LCP, (c) input linearly polarized, (f) input elliptically polarized.

the GP lenses employed, as described in Section 3 [20]. Obviously, changing the input wavelength requires rescaling the telescope distances, since different wavelength implies a different focal length of the GP lenses, thereby varying the doublet focal lengths $f_{\pm}$. The simplest choice is to calculate the new separation h between doublets that ensures the dual-polarization imaging condition on the same plane, while maintaining the doublet size d . The new image-plane position s' and magnification ratio m^2 both change with wavelength. Within the considered spectral range, the magnification ratio m^2 varies from 10.1 to 22.5, as h is adjusted from 137 mm to 174 mm. We have verified that within the 600–750 nm range, the system is indeed a telescope ($C = 0$ of the ABCD matrix).

Finally, we analyzed the telescope sensitivity to the spectral bandwidth by considering a quasi-monochromatic beam with 10 nm spectral width. Maximum image-plane deviations of 0.9% for RCP and 10.5% for LCP were obtained with respect to the monochromatic nominal wavelength of 632.8 nm, considering the linear relation $\lambda f_G = 58.87$ [nm·m] of the employed GP lenses [26]. The magnification ratio is affected by up to 1.3%.

Let us note that, even when considering the exact design wavelength of 632.8 nm, the imaging quality of the proposed telescope relies strongly on the performance of the two GP lenses employed. As shown in the experimental images, non-focused parasite light appears because of the non-100% polarization-conversion efficiency, thus reducing the image contrast.

6. Conclusions

In summary, this work presents the design and experimental implementation of a dual-polarization telescope composed of two lens doublets, each consisting of a refractive lens and a GP lens. This telescope system achieves, on a common imaging plane, different magnifications for the orthogonal circular polarization components; namely, one is

magnified whereas the other is demagnified. The telescope performance relies on the bifocal polarization-dependent properties of the constituent lens doublets, which were derived using the ABCD ray matrix formalism, and on the symmetry properties and RCP/LCP conversion of GP lenses.

A first key step in the design of the lens doublet was to provide two real foci, thus surpassing the limitation of GP lenses alone, which exhibit the same focal length but with opposite sign for orthogonal circular polarizations. In contrast, the lens doublet provides axial polarization splitting with two positive and different focal lengths that depend on the focal lengths of the refractive and GP lenses, and on their separation d . A second key step was to experimentally verify this splitting property and the locations of the back and front focal planes versus d , using a commercial GP lens combined with a regular lens. It was verified by observing the spacing between the diffraction orders generated by a diffraction grating, a simple method that is not affected by the axial location of the principal planes.

A third key step was the design and experimental validation of a Keplerian telescope configuration based on two such bifocal lens doublets. In this optical system, the two real back foci of the first lens doublet are coupled to the two front foci of the second lens doublet. Here, the symmetry properties and RCP/LCP conversion of GP lenses is crucial. In this configuration, the telescope performs with an inverse magnification for one circular polarization component with respect to the orthogonal polarization. The conditions for this telescope to yield the two images of a near object on the same output image plane were derived and experimentally verified. Since it relies on GP lenses, the proposed telescope is intended to operate at the design wavelength and in paraxial approximation. Besides, the system can be readily redesigned for any other wavelength within the spectral range where the GP lens exhibits high polarization-conversion efficiency. In these conditions, there is negligible efficiency loss due to zero-order stray light or background noise at the image plane.

In the proof-of-concept of this novel design we have used an iris diaphragm to reduce the impact of aberrations in the GP lenses and

operate in first-order optics. Thus, the system has a limited angular field of view. Enlarging the field of view generally benefits the application scenarios. Here the GP lenses are the limiting components. Therefore, strategies should focus on improving the limited field of view of these lenses. Implementing compensating layers, as it was done in liquid-crystal displays [28], could be considered. Also, recent advances have demonstrated the use of spatial multiplexing of nanograting that enable LC lenses to achieve wide field-of-view imaging with high efficiency and compactness [29].

This study might open new paths for designing polarization-sensitive optical systems, such as digital holographic systems [30] or common-path polarization interferometers [31] which can employ bifocal lens doublets to axially split the circular polarization components into different focal spots. For instance, producing co-planar magnified and demagnified replicas with a tunable scaling factor provides a natural common-path building block for radial shearing interferometry [32], which can be real-time when detected with a polarization camera [33]. In imaging systems, achieving variable magnification typically requires mechanical zoom lenses with moving elements. By linking magnification to the polarization state, instantaneous non-mechanical optical zoom or simultaneous multi-scale imaging can be achieved. Thus, combined with metalens arrays could be applied to polarization-sensitive integral imaging 3D displays [34]. Combined with polarization optical modulators —like fast liquid-crystal or electro-optic modulators— this system becomes a passive fast-zoom system without mechanical changes, alternative or complementary to other tunable technologies like liquid lenses employed in imaging systems [35,36] and in optical communications [37]. Fields like machine vision, autonomous driving or surgical imaging could benefit from this high-speed polarization-controlled switching between a wide-angle view and a high-magnification telephoto view.

CRedit authorship contribution statement

Santiago Ríos-Álvarez: Writing – review & editing, Validation, Software, Investigation, Formal analysis. **David Marco:** Writing – review & editing, Visualization, Supervision, Methodology, Formal analysis. **Ignacio Moreno:** Writing – review & editing, Validation, Project administration, Conceptualization. **María del Mar Sánchez-López:** Writing – review & editing, Writing – original draft, Supervision, Project administration, Methodology, Conceptualization.

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Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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