

Analysis of multiple internal reflections in a parallel aligned liquid crystal on silicon SLM

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Abstract: Multiple internal reflection effects on the optical modulation of a commercial reflective parallel-aligned liquid-crystal on silicon (PAL-LCoS) spatial light modulator (SLM) are analyzed. The display is illuminated with different wavelengths and different angles of incidence. Non-negligible Fabry-Perot (FP) effect is observed due to the sandwiched LC layer structure. A simplified physical model that quantitatively accounts for the observed phenomena is proposed. It is shown how the expected pure phase modulation response is substantially modified in the following aspects: 1) a coupled amplitude modulation, 2) a non-linear behavior of the phase modulation, 3) some amount of unmodulated light, and 4) a reduction of the effective phase modulation as the angle of incidence increases. Finally, it is shown that multiple reflections can be useful since the effect of a displayed diffraction grating is doubled on a beam that is reflected twice through the LC layer, thus rendering gratings with doubled phase modulation depth.

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1. Introduction

Parallel-aligned liquid-crystal spatial light modulators (PAL-LC-SLM) may be ideally regarded as pixelated linear retarders with tunable retardance, useful for applications where pure phase modulation is a key feature [1]. These include phase or amplitude control in ellipsometric systems [2], shaping of femto-second pulses [3], generation of diffractive optical elements [4] and wavefront sensors [5], among others. PAL-SLMs ideally offer pure phase modulation when illuminated with light that is linearly polarized parallel to the LC director. They are usually designed to produce at least 2π phase modulation at the operating wavelength. Their phase response versus addressed gray level is sometimes linearized at the electronic driver level. In addition, either normal or small oblique angles of incidence are usually recommended by manufacturers for reflective liquid crystal on silicon (LCoS) devices.

In practice, however, the response of PAL-SLMs as ideal programmable retarders is deteriorated by several secondary effects. First, the pixelated structure introduces diffraction losses [6]. In LCoS devices these losses have been reduced thanks to their better fill factor, since pixelated electrodes are required only on one side of the panel. However, LCoS-SLMs are affected by other drawbacks like non-uniformities in the backplane [7,8], and fringing field effects, which become important due to their increased spatial resolution [9]. Temporal phase fluctuations or flickering due to digital addressing schemes might also be present in many LCoS devices [10]. All these effects deteriorate their effective modulation, and they must be considered if optimal diffraction efficiency and accurate measurements are required.

In this work we analyze another effect that might substantially modify the optical modulation: Fabry-Perot (FP) multiple-beam interference. Although FP effects have found uses in LC devices to build tunable filters [11], they are usually secondary effects that degrade their performance [12]. When the LC-SLM is illuminated with light of a broadband spectrum, FP effects can be easily noticed as rapid oscillations in the transmitted spectrum [13]. Using monochromatic light, FP effects become also apparent as intensity oscillations versus applied voltage when the input beam is linearly polarized along the LC director [14].

In all the aforementioned works, the devices were transmissive PAL-LC modulators illuminated under normal incidence. However, LCoS devices operate in reflection, and therefore the FP interference effect adopts the Gires-Tournois interferometer configuration [15], with a first low reflective layer, and a second highly reflective layer.

LCoS devices are sometimes illuminated under angles of incidence other than normal incidence. In this situation the FP interference condition is substantially affected. In addition, achieving phase modulation regimes substantially higher than 2π is interesting to produce unusual chromatic applications [16]. Since SLMs with such a very large phase modulation are not available, we recently proposed the use of visible light with a PAL-LCoS designed to be operated with infrared light (IR) [17]. In such cases, operating far from the design conditions, multiple reflection interference becomes even more relevant.

Therefore, the goal of this paper is to analyze FP interference effects in LCoS devices, and their implications in their phase-only modulation, including changing the angle of incidence. The observed effects are: 1) undesired coupled amplitude modulation, 2) a non-linear phase modulation, 3) a fraction of unmodulated light, and 4) a phase depth reduction as the angle of incidence increases. We propose a simple physical model that quantitatively accounts for all the observed effects with a reduced set of optical parameters.

In addition, the diffraction efficiency losses due to FP effects are analyzed. We will check that, for small diameter beams illuminating the device under a large-enough angle of incidence, multiple reflected beams can be spatially separated. In this situation, it will be shown how the effect of a displayed diffraction grating is doubled on a beam that has been reflected twice through the LC layer. We will show that this effect can render doubled phase modulation diffractive gratings.

The paper structure is the following: In section 2 the LCoS device is presented and interference effects are experimentally characterized for different wavelengths and angles of incidence. In section 3, the FP simplified physical model is presented. In section 4 the theoretical predictions are compared to the experimental modulation curves in order to obtain the parameters that fit the model. Section 5 includes an analysis of the phase-modulation characteristics, and section 6 includes experimental diffraction results with binary and blazed gratings are displayed on the device. The consequences of FP effects on the diffraction efficiency are discussed. The provided experimental results show that the beam reflected twice by the LC layer presents a double diffraction grating effect in comparison to the main first reflection. Finally, in section 7 conclusions of the work are presented.

2. Experimental evidence of multiple reflections

A commercial LCOS device was used in this work. It is a PAL-LCoS-SLM model X10468-08 from Hamamatsu, with 792×600 pixels, $20 \times 20 \mu\text{m}^2$ pixel size and video-rate operation (60 Hz). Although it was designed to work in the 1000 nm to 1500 nm range, we use visible light in order to achieve a very large phase modulation. Such large phase modulation is of interest to produce diffractive elements with wavelength compensated efficiency [16] or wavelength compensated dispersion [17]. Nevertheless, similar FP phenomena as presented here have been observed in other devices from the same series designed to operate with visible light.

Figure 1 shows the layer structure of the LCoS device described by the manufacturer. It consists on the following series of dielectric and conductor layers:

- i. A protective glass (thick layer),
- ii. A transparent electrode,
- iii. An alignment film,
- iv. The liquid crystal layer,
- v. An alignment film,
- vi. An aluminum mirror (or dielectric mirror in other models),

vii. The silicon substrate.

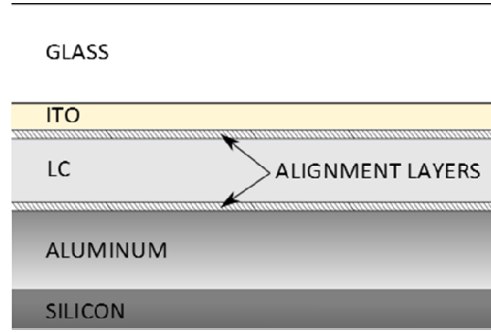


Fig. 1. Layer structure of the LCoS display.

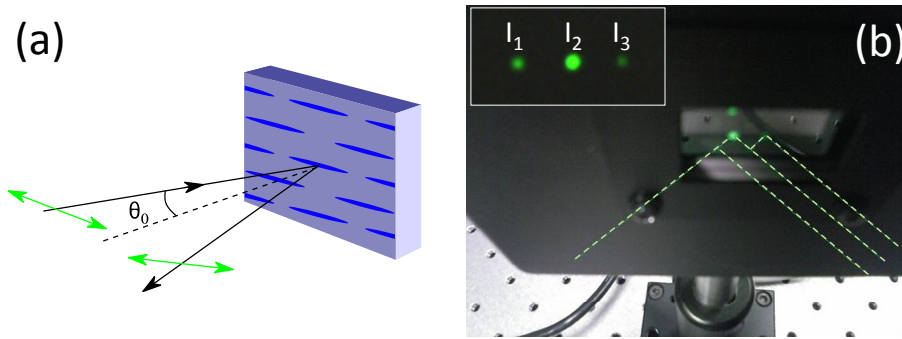


Fig. 2. (a) Illumination scheme, where incident polarization is selected parallel to LC director. (b) Picture of the LCoS-SLM illuminated with $\theta_0 = 45^\circ$ and $\lambda = 568$ nm, where light scattered at the reflecting surfaces can be noticed. Incoming and emerging light are outlined by dashed lines. The reflected beams captured by a CCD camera are shown in the upper left inset box.

Figure 2(a) depicts the illumination scheme suggested by the manufacturer. The device is illuminated by a horizontally linearly polarized TM beam with angle of incidence θ_0 . The linear polarization plane coincides with the LC director axis. Thus, extraordinary waves are excited inside the LC medium. At normal incidence, multiple reflected beams overlap. However, if the beam width is narrow enough and the angle of incidence is large enough, multiple reflected beams can be spatially separated. Figure 2(b) shows evidence of these multiple reflections. Here a non-expanded laser beam illuminates the LCoS screen with $\theta_0 = 45^\circ$. The reflected beam is captured with a CCD camera. Three main reflections, denoted as I_1 , I_2 and I_3 , are clearly observed (the CCD image is shown in the inset of Fig. 2(b)). The first reflection (I_1) is generated on the front surface of the protective glass. The most intense central reflection (I_2) is the main beam, which is reflected at the back surface, thus traversing the LC layer. Finally, the third reflected beam (I_3) is generated after a reflection inside the protective glass, and another reflection on the LC layer. Lines indicating these beams are drawn on the photograph in Fig. 2(b) to illustrate them.

The intensity of these reflected beams was measured versus the addressed gray level. Three wavelengths from an Ar-Kr laser were used: $\lambda = 488$ nm, $\lambda = 568$ nm, and $\lambda = 647$ nm. Two different angles of incidence were selected. In Figs. 3(a), 3(b), and 3(c) the angle was small, approximately $\theta_0 = 15^\circ$. Here, although the three reflected beams had a slight lateral separation, it was difficult to spatially filter them. In consequence, they were measured together and the measurements represent the addition of the intensities of the three reflected beams. On the contrary, Figs. 3(d), 3(e) and 3(f) correspond to $\theta_0 = 45^\circ$, where the three beams have enough lateral separation to be spatially filtered, and be measured independently.

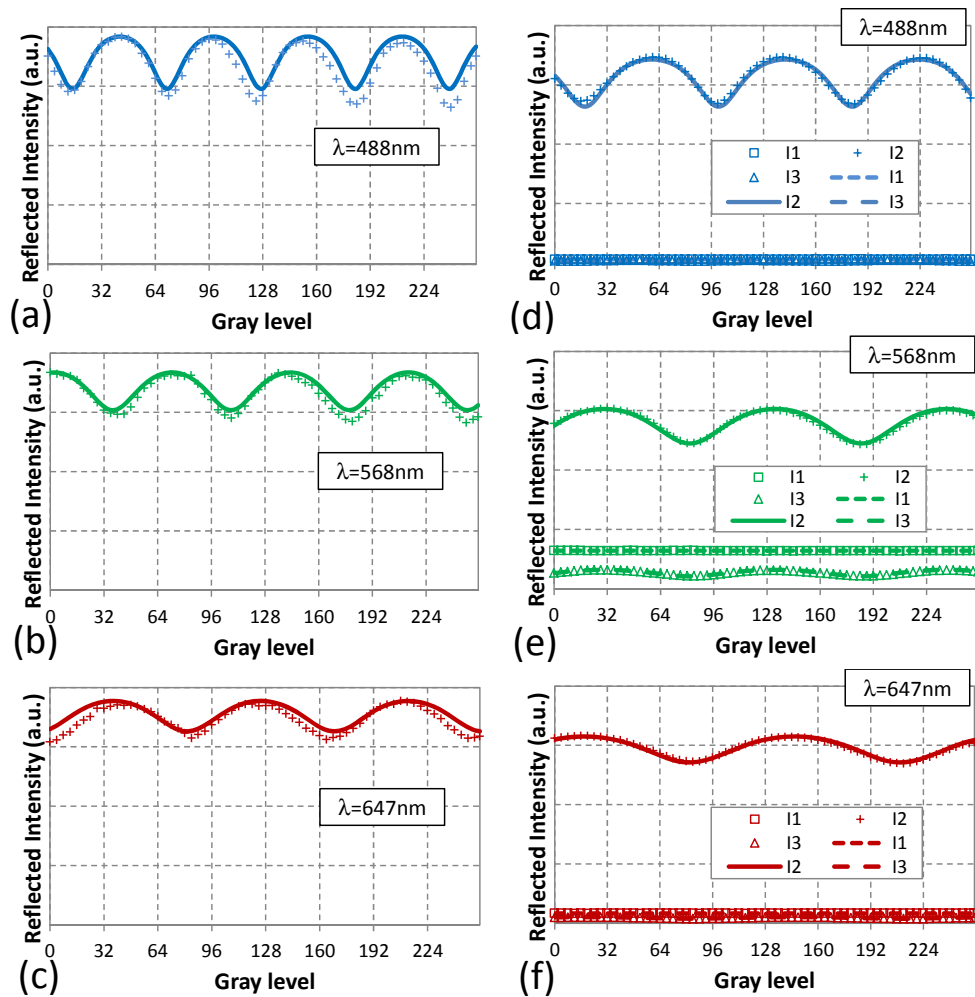


Fig. 3. Reflected intensity versus gray level for (a) 488 nm, (b) 568 nm, (c) 647 nm for $\theta_0 = 15^\circ$. Reflected intensities I_1 , I_2 , and I_3 versus gray level for $\theta_0 = 45^\circ$ for (d) 488 nm, (e) 568 nm, and (f) 647 nm. Lines show the corresponding numerical curves after fitting the simplified physical model.

These figures highlight a second multiple reflection effect originated at the LC layer. The observed characteristic oscillation in the output intensity versus the gray level denotes the presence of FP interference, similar to that observed in [14] for a transmission display under normal incidence. Since the LC layer is very narrow, multiple reflected beams in this layer overlap and interference cannot be avoided.

Two interesting effects are noticeable in Fig. 3. First, the number of oscillations decreases as the wavelength increases. For $\theta_0 = 15^\circ$, more than 4.5 oscillations are observed for 488 nm (Fig. 3(a)), 3.5 oscillations for 568 nm (Fig. 3(b)), and 3 oscillations for 647 nm (Fig. 3(c)). This is in agreement with the smaller phase obtained for larger wavelengths. Second, for $\theta_0 = 45^\circ$, the number of oscillations for the I_2 main reflected beam is reduced for the three wavelengths. Now only 3 oscillations are observed for 488 nm (Fig. 3(d)), 2.5 oscillations for 568 nm (Fig. 3(e)), and 2 oscillations for 647 nm (Fig. 3(f)). The first reflected beam (I_1) shows a constant intensity insensitive to changes in the gray level, thus confirming that it is generated at the most external air-glass reflection. The third reflection (I_3) is less intense than the second one (I_2), but oscillates in the same manner.

In order to explain this modulation behaviour, a simple physical model that considers a three layer reflective structure is proposed. We will show that such a model quantitatively describes the observed phenomena, with a reduced set of optical parameters. The solid lines in Fig. 3 show the predictions of the proposed model, which will be explained next.

3. Simplified model of the LC-device layer structure

As expected, the main FP effect is produced at the birefringent LC-layer, where the optical path lengths are modulated by the driving voltage. Therefore, intermediate electrode and substrate layers will be discarded in the simple model proposed here. Therefore, we will consider the LCoS device as a three layer medium with the following layers:

1. A thick layer of protective glass (with refractive index n_1),
2. A thin layer of liquid crystal (with refractive index n_2),
3. A highly reflective aluminum layer (with refractive index n_3),

while the external medium (referred to as 0) is air.

Figure 4(a) depicts the outline of the principal reflected beams (I_1 , I_2 and I_3). The incident beam, with intensity I_i , impinges onto the LCoS device at an angle of incidence θ_0 . The first beam is reflected on the air-glass interface (point P_1 in Fig. 4(a)), with intensity I_1 . The transmitted beam, with a refraction angle θ_1 , propagates through the glass and reaches the LC layer (point Q_1). Here a beam is reflected after traversing the thin LC layer. This reflection is affected by FP interference, which depends on the addressed gray level. The reflected beam reaches the glass-air interface (point P_2), and produces the second beam (main modulated beam), of intensity I_2 . But another beam is reflected back at point P_2 in this interface and, after a second reflection through the LC layer (point Q_2), ends up producing the third reflected beam (point P_3), of intensity I_3 .

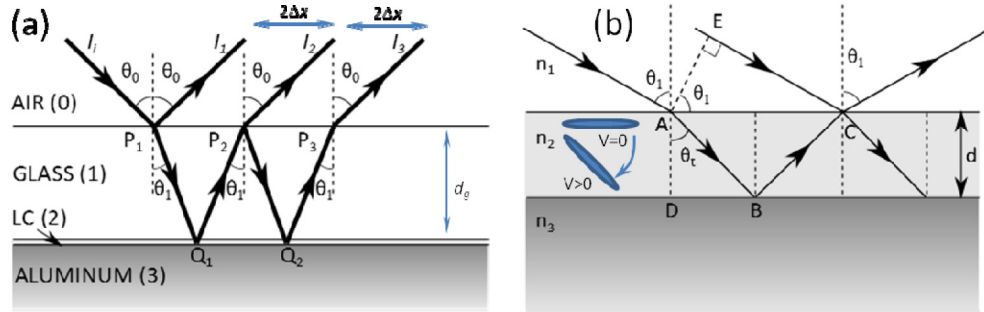


Fig. 4. (a) Simplified layer structure and main transmitted/reflected beams. (b) Schematic of the interference condition at the LC layer. Voltage dependent LC director realignment in this layer is indicated

Note that, from Fig. 4(a), the lateral displacement, $2\Delta x$, between consecutive reflected beams can be calculated through geometrical considerations to be:

$$2\Delta x = \frac{2d_g \sin \theta_0}{\sqrt{n_1^2 - \sin^2 \theta_0}}, \quad (1)$$

where d_g denotes the width of the glass layer. Therefore, the condition to avoid overlapping of these multiple reflected beams (I_1 , I_2 and I_3) is: that

$$\Delta x > \frac{w}{2 \cos \theta_0}, \quad (2)$$

being w the waist of the laser beam. Thus, the following limit to w can be derived to obtain spatially separated beams can be retrieved:

$$w < \frac{2d_g \sin \theta_0 \cos \theta_0}{\sqrt{n_1^2 - \sin^2 \theta_0}}, \quad (3)$$

For instance, for $\theta_0 = 45^\circ$, and assuming $d_g = 1$ mm and $n_1 = 1.5$, Eq. (3) indicates that the laser beam on the LCoS screen must have a width $w < 0.7$ mm, which is in good agreement with the observed experiment.

Figure 4(b) illustrates in more detail the interference that occurs at the LC layer (i.e. the reflection at point Q_1 in Fig. 4(a)). Optical modulation is produced here by realignment of the LC director. Let us point out that since the LC layer is an anisotropic medium, the observed FP effects are noticeable only for the polarization component parallel to the LC director, which excites the extraordinary wave inside such medium. Therefore, refraction inside the LC layer does not follow Snell's law, and formulations for analyzing reflection at anisotropic layers [18] are required. However, as we shall show next, a good approximation of the observed FP effect can be obtained by considering the LC media as isotropic with a refractive index n_2 that changes under the action of the applied voltage (it corresponds to the effective extraordinary index). Since the LC layer is so thin, deviations from Snell's law are expected to have relatively small impact. Note that such an approximation has been already successfully applied in many other physical models of LC devices [19–21].

The overall reflection coefficient of the LC three layer structure (i.e. the effective reflection coefficient occurring at Q_1 in Fig. 4(a)) reads [22]:

$$\rho_{FP} = \frac{\rho_{12} + \rho_{23} \exp(ik_0 \Delta L)}{1 + \rho_{12} \rho_{23} \exp(ik_0 \Delta L)}, \quad (4)$$

where ρ_{12} is the reflection coefficient from glass to LC, ρ_{23} is the reflection coefficient from LC to the reflective layer, and ΔL is the optical path difference between trajectories ABC and EC in Fig. 4(b). The related phase shift $k_0 \Delta L$ is given by:

$$k_0 \Delta L = \frac{4\pi}{\lambda_0} dn_2 t, \quad (5)$$

where k_0 is the wave number, λ_0 is the vacuum wavelength, n_2 is the LC layer refractive index and d is its thickness. If the LC layer were isotropic, from the geometry of Fig. 4(b), the optical path difference would be $\Delta L = n_2 ABC - n_1 EC = 2dn_2 \cos \theta_t$, being θ_t the transmitted angle inside the LC layer. However, since it is an anisotropic layer, a parameter $t \leq 1$ is used instead, with no direct relation to the angle θ_t . This parameter will be fitted to the experimental data. Nevertheless, note that a relevant dependence of the phase shift in Eq. (5) on the angle of incidence θ_0 arises through this parameter. For normal incidence t can be assumed to be equal to one, and to significantly reduce its value as θ_0 increases.

4. Modulation characterization and fitting results

In order to characterize the observed FP effects, and obtain the model parameters, we evaluate the data shown in Figs. 3(d)-3(f), corresponding to $\theta_0 = 45^\circ$. These curves correspond to the three following paths, shown in Fig. 4(a):

Beam 1: First reflection at the air-glass interface whose intensity is expected to be a constant value given by:

$$I_1 = I_i |\rho_{01}|^2, \quad (6)$$

where ρ_{01} is the reflection coefficient at the air-glass interface, which we assume to be a real-valued number.

Beam 2: Transmission at the air-glass interface, reflection at the LC layer and transmission from glass to air. The beam intensity is given by:

$$I_2 = I_i |1 - \rho_{01}^2|^2 |\rho_{FP}|^2, \quad (7)$$

where the overall LC layer reflection coefficient ρ_{FP} was given in Eq. (4).

Beam 3: Transmission at the air-glass interface, reflection at the LC layer, reflection at the glass-air interface, a second reflection at the LC layer, and transmission through interface glass-air. Its corresponding intensity is expected to be

$$I_3 = I_i |1 - \rho_{01}^2|^2 |\rho_{01}|^2 |\rho_{FP}|^4, \quad (8)$$

Note that the above relations imply no interference between the first, second and third reflected beams. This can be assumed in our system as long as the laser beam is maintained with a small diameter and the incidence is not selected normal.

Using Eqs. (3)-(5), ρ_{01}^2 and $|\rho_{FP}|^2$ are calculated for each addressed gray level as follows:

$$\rho_{01}^2 = \left(1 + \frac{I_2}{\sqrt{I_1 I_3}} \right)^{-1}, \quad (9)$$

and

$$|\rho_{FP}|^2 = \frac{I_2}{I_1} \frac{\rho_{01}^2}{(1 - \rho_{01}^2)^2}. \quad (10)$$

Let us remark that with this procedure the determination of ρ_{01} and ρ_{FP} is independent of the incident beam intensity I_i and relies entirely on the intensity measurement of the three consecutive reflected beams (I_1 , I_2 and I_3). Moreover, isolating the FP interference term at the LC layer from undesired terms avoids changing the measurement conditions.

The results derived from the experimental data in Figs. 3(d)-3(f) are shown in Figs. 5(a)-5(c). $|\rho_{01}|^2$ takes a constant value independent of the gray level. On the contrary, $|\rho_{FP}|^2$ exhibits interference oscillations. These experimental values are used to obtain the physical parameters of the model through Eqs. (4) and (5).

The model parameters are:

- Reflection coefficients ρ_{12} and ρ_{23} , which are assumed to be independent of θ_0 , but dependent on the wavelength. As initial search values in the fitting procedure, values close to 0.1 and 0.9 were selected for ρ_{12} , and ρ_{23} , respectively.
- Parameter t , which is assumed to be wavelength independent, but depends on θ_0 ; $t \approx 1$ is expected for small θ_0 , and smaller values are expected as θ_0 increases.
- The variation with the addressed gray level of the phase shift $k_0 \Delta L$ given in Eq. (5). For convenience, this phase shift will be expressed as:

$$k_0 \Delta = \beta_0 t + \Delta \beta(g) t, \quad (11)$$

where $g \in [0, 255]$ denotes the addressed gray level, β_0 is the phase shift for $g = 0$, and $\Delta \beta(g)$ represents the birefringence variation with g .

In general, LC displays provide a phase modulation with a non linear relation with g . This reduces the efficiency of displayed diffractive elements, and compensation at a software level is required to linearize the phase levels. However, some manufacturers, as it is here the case, provide compensation at the hardware level, so the relation between phase levels and

addressed gray levels is linear. This compensation is valid for normal incidence. Therefore, $\Delta\beta(g)$ in Eq. (11) can be assumed as:

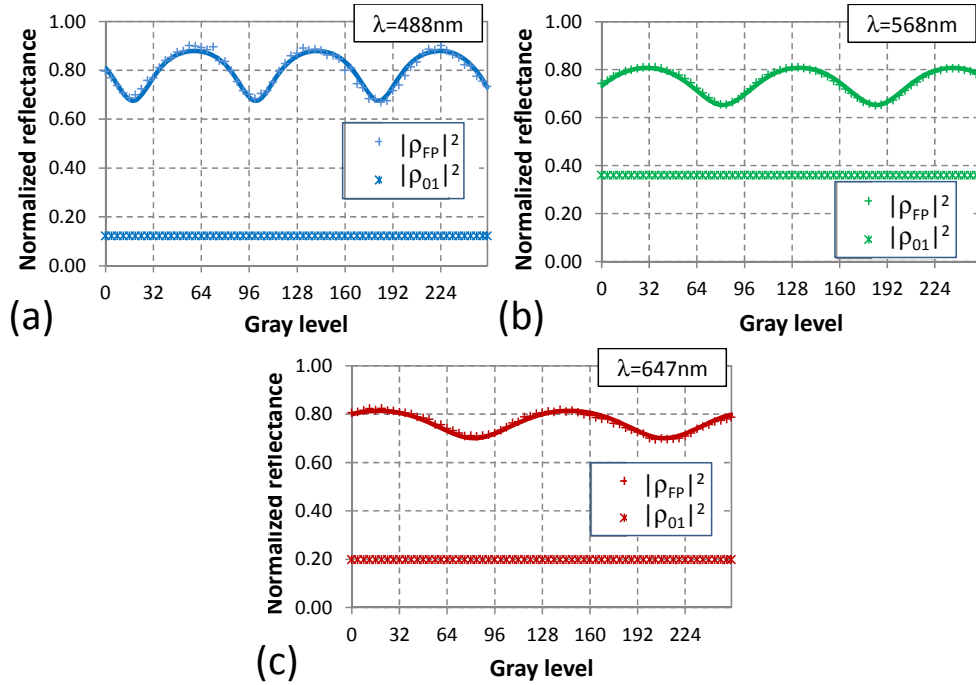


Fig. 5. FP and air-glass interface reflectance corresponding to measurements (solid lines) and fitted data (crosses and asterisks) for wavelengths: (a) 488 nm, (b) 568 nm, and (c) 647 nm.

$$\Delta\beta(g) = \Delta\beta_{\max} \left(1 - \frac{g}{255} \right), \quad (12)$$

where $\Delta\beta_{\max}$ is the maximum phase change due to the LC birefringence variation. The maximum phase modulation the LCoS device can provide is regarded to be $2\Delta\beta_{\max}$. Therefore, the remaining two parameters required to fit the physical model are β_0 and $\Delta\beta_{\max}$, which both depend on the wavelength and take larger values for shorter wavelengths.

It is very important to note that, while in an isotropic medium β_0 and $\Delta\beta_{\max}$ would not depend on the angle of incidence θ_0 , they do depend on it in an anisotropic medium since they are directly related to the effective extraordinary index (which is not constant with the angle of incidence). Thus, we expect to obtain close (but not the same) values for these parameters at different angles of incidence. For the sake of simplicity, we will keep $\Delta\beta_{\max}$ independent of the angle of incidence, but small variations on β_0 will be allowed.

Therefore, the experimental curves in Figs. 5(a), 5(b) and 5(c) corresponding to $\theta_0 = 45^\circ$, were fitted to numerical calculations based on Eqs. (4), (5), (11), and (12), by adjusting parameters ρ_{12} , ρ_{23} , t , β_0 and $\Delta\beta_{\max}$ with the above mentioned constrains. The results of this fitting process are shown in Fig. 5 as solid curves that overlap the experimental data. Excellent agreement is found in all cases. Table 1 shows the fitting parameters retrieved for each wavelength. As expected, values for ρ_{12} are much lower than ρ_{23} in all cases. The birefringence $\Delta\beta_{\max}$ diminishes with increasing λ and $t = 0.683$.

Next, the same set of parameters are used in Eqs. (3), (4), and (5) to match the intensity of the three reflected beams measured at $\theta_0 = 45^\circ$ which are shown in Figs. 3(d), 3(e), and 3(f). The fitting results are depicted as solid lines in these figures which, again, keep an excellent agreement with the experimental data.

Table 1. Values obtained after the fitting process for the reflection coefficients and birefringence parameters at the three calibration wavelengths.

λ	ρ_{01}	ρ_{12}	ρ_{23}	$2\Delta\beta_{\max}$	$2\beta_0(\theta_0 = 45^\circ)$	$2\beta_0(\theta_0 = 15^\circ)$
488 nm	0.122	0.272	0.894	9.1π	16.43π	16.43π
568 nm	0.360	0.171	0.860	7.2π	11.17π	10.82π
647 nm	0.198	0.135	0.873	5.8π	9.22π	9.03π

Finally, the same values are applied to predict the modulation for $\theta_0 = 15^\circ$ observed in Figs. 3(a), 3(b) and 3(c). In these predictions, $t = 1$ was selected and simply β_0 was readjusted to provide the best fit. The newly retrieved β_0 values, also displayed in Table 1, are slightly different from the values at $\theta_0 = 45^\circ$. The numerical simulation, included in Figs. 3(a), 3(b) and 3(c) as solid lines, accounts excellently for the measured data, thus confirming the predictions of the proposed physical model. A quantitative estimation of this agreement is given in Table 2. The mean square error is calculated for the angles of incidence of 15° and 45° , between each experimental data and its corresponding simulated data, and for all three measured wavelengths. For $\theta_0 = 15^\circ$, the error is calculated for the curves in Figs. 3(a), 3(b) and 3(c), while for $\theta_0 = 45^\circ$, it is calculated only for the I_2 curve, since it is the only one that shows the oscillatory behavior. As expected the error is very low for the incidence of 45° , since it is being used to fit the model parameters, and it is greater for incidence of 15° , although providing rather good agreement.

Table 2. Mean square error between experimental data and model predictions for the data presented in Fig. 3.

λ	$\theta_0 = 15^\circ$	$\theta_0 = 45^\circ$
488 nm	12.4%	3.7%
568 nm	7.5%	2.0%
633 nm	7.9%	1.2%

5. Phase modulation characteristics

Once the physical model has been set, it can be used to predict the phase modulation characteristics of the device. Note that the phase modulation $\Delta\phi(g)$ is given by the phase difference between the complex coefficient ρ_{FP} in Eq. (1) for different gray levels, i.e.,

$$\Delta\phi(g) = \arg\{\rho_{FP}(g)\} - \arg\{\rho_{FP}(g=0)\}. \quad (13)$$

At normal incidence, and ignoring FP interference effects, the maximum phase modulation can be approached to be equal to the maximum birefringence variation, i.e., $\Delta\phi(g) \cong 2\Delta\beta(g)$. However, phase modulation significantly reduces when the angle of incidence increases up to 45° .

Figure 6 shows as solid curves the phase modulation $\Delta\phi(g)$ that is deduced from Eqs. (4) and (13) at $\theta_0 = 45^\circ$. Non-linearities in the phase evolution can be observed as a consequence of the FP interference. Dashed lines in Fig. 6 represent $2\Delta\beta(g)$, with maximum values $2\Delta\beta_{\max} = 9.1\pi$, 7.2π , and 5.8π for wavelengths 488 nm, 568 nm, and 647 nm, respectively, which very well match the phase modulation observed in Figs. 3(a), 3(b) and 3(c) for $\theta_0 = 15^\circ$.

These results confirm that the optical path difference ΔL in Eq. (2) depends on parameter t . The reduction in parameter t from $t = 1$ at normal and small incidence ($\theta_0 = 15^\circ$), to $t = 0.683$ at $\theta_0 = 45^\circ$ explains phase modulation depth reduction that is observed for greater incidence angles. This is a relevant conclusion of our work. This phase modulation reduction with θ_0 was already noticed in [23] for a twisted nematic LCoS display, and recently in [24] for a PAL-LCoS display. The model here developed provides a physical explanation.

To better understand the physical insights of this phenomena, we can assume that $\rho_{12} \ll \rho_{23}$ and $\rho_{12}\rho_{23} \ll 1$ to approximate the complex modulation in Eq. (4) as:

$$\rho_{FP} \cong \rho_{23} \exp(i2k_0 d n_2 t) \propto \rho_{23} \exp(i2\Delta\beta_{\max} t). \quad (14)$$

Since t is related to θ_0 , it can be understood that phase modulation should directly depend on θ_0 . If the LC layer were isotropic, then $t = \cos\theta_0$, and the transmitted (θ_t) and incident (θ_0) angles would be directly linked through Snell's law. Therefore they would increase or decrease together. The maximum phase modulation $2\Delta\beta_{\max}t$ should get its maximum value, $2\Delta\beta_{\max}$, at normal incidence where $\theta_t = \theta_0 = 0^\circ$ and $t = 1$, and it should decrease with larger angles of incidence. Since the LC layer is anisotropic, an exact description of the t factor still needs to be performed. Nevertheless, the experimental results shown here indicate that maximum phase modulation is produced at normal incidence whereas it is reduced by a factor $t < 1$ if greater angles of incidence are used.

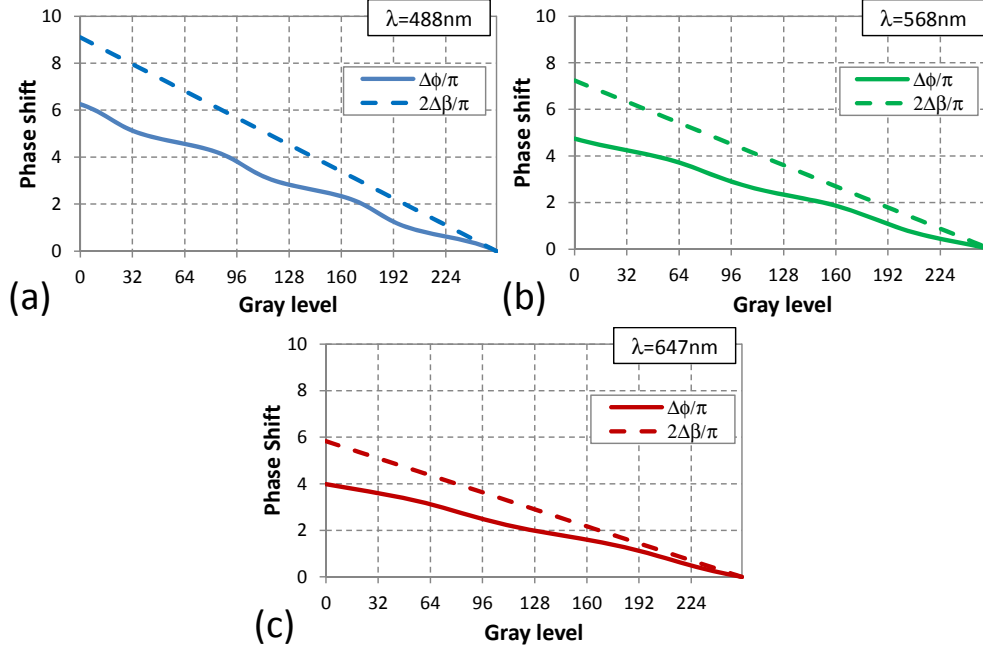


Fig. 6. Estimated phase modulation (solid curve) and birefringence variation (dashes) for wavelengths: (a) 488nm, (b) 568 nm, and (c) 647nm.

6. Effects on phase-only diffraction gratings

In this final section, we incorporate the above described multiple reflection effects in the modulation diffraction efficiency of the display, and analyze how they affect encoded phase-only diffraction gratings. Results with binary-phase gratings and with continuous blazed-phase gratings are discussed. For these results, angle of incidence $\theta_0 = 45^\circ$, wavelength $\lambda = 568$ nm and the main modulated reflected beam (I_2) are only considered. Experiments have been conducted with other wavelengths and normal incidence in [17].

As it has been shown, interference effects produce non-desired behaviour as coupled amplitude modulation and phase non-linearities, which reduce the diffraction efficiency. After [25], the following general expression can be derived for the amplitude of the m^{th} diffraction order generated by a displayed linear phase-only diffraction grating:

$$c_m = \sum_{n=0}^{N-1} \rho_{FP}(g_n) \sin\left(\frac{m\pi w}{P}\right) \frac{1}{m\pi w} \exp\left(-i \frac{2\pi m n w}{P}\right), \quad (15)$$

where P is the period of the grating, N is the number of steps in each period, w is the width of each step, so $Nw = P$, and $\rho_{FP}(g_n)$ is the constant reflection coefficient defined in Eq. (4), at

level n , which is addressed by gray level g_n . The diffraction efficiency at each diffraction order m can be calculated as $\eta_m = |c_m|^2$.

Special situations can be obtained with binary-phase gratings ($N = 2$) and blazed gratings, which allow to easily identify the maximum phase level achieved in phase regimes [26]. A binary phase grating produces a diffraction pattern where the zero (DC) order is cancelled whenever the phase difference between the two phase levels is $\pi, 3\pi, 5\pi, \dots$. On the contrary, if the phase difference is $0, 2\pi, 4\pi, \dots$, there is not an effective phase difference, and no diffraction orders arise except the zero order.

For blazed gratings, light gets progressively diffracted to higher diffraction orders as the maximum phase modulation ($\Delta\phi_{\max}$) increases. If $\Delta\phi_{\max} = 2\pi$, all the energy is diffracted to the first diffraction order; if $\Delta\phi_{\max} = 4\pi$, then all light is diffracted onto the second diffraction order, etc. On the contrary, if $\Delta\phi_{\max} = \pi$, most of the energy is split between the zero and first diffracted orders, which have equal intensity. For $\Delta\phi_{\max} = 3\pi$, most of the energy is now split between the first and second diffraction orders again with the same intensity [26].

In Fig. 7 the diffraction efficiency of the second reflected beam (I_2) is simulated for $\lambda = 568$ nm and $\theta_0 = 45^\circ$, taking into account the coupled amplitude modulation (I_2 curve in Figs. 3(e)) and the effective phase modulation $\Delta\phi$ (Figs. 6(b), solid curve). In Fig. 7(a), the efficiencies at the zero and first diffracted orders, $\eta_0 = |c_0|^2$ and $\eta_1 = |c_1|^2$, are calculated for a binary phase grating as one gray level is increased while the other is set to $g = 0$. An effective reduction of the overall maximum efficiency due to losses induced by the ρ_{FP} magnitude is observed. Nevertheless, whenever $\Delta\phi$ reaches an odd integer value of π , the zero order is cancelled and an approximate 40.5% efficiency for η_1 relative to the maximum value η_0 is reached. Note the non-symmetric features of the efficiency curves, caused by the non-linearities in the phase modulation.

Figure 7(b) depicts the 0^{th} , 1^{st} and 2^{nd} diffraction order intensities for a blazed grating, with linear gray levels increasing from zero to a maximum value g , and diffraction efficiencies are evaluated as a function of g . Therefore, the maximum phase increases from zero up to a maximum value $\Delta\phi(g)$. In this case, the intensity maxima do not reach 100% and the minima do not extinguish for phase values integer multiples of 2π , as it is expected for an ideal blazed grating. This is a consequence of the amplitude modulation and the phase non-linearities introduced by the interference effect. Nevertheless, it can be appreciated that most of the energy is diffracted onto the first diffraction order when the maximum phase reaches 2π (red curves), and onto the second diffracted order where it reaches 4π (green curves).

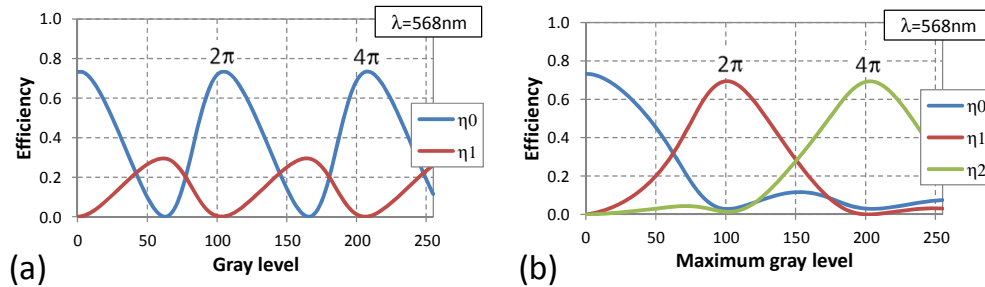


Fig. 7. Diffraction efficiency (η_m) at the m -th order diffraction efficiency versus gray level for (a) binary grating, and (b) blazed grating corresponding to the reflectance obtained for 568 nm.

Figure 8 shows CCD images of the binary grating experimental diffracted orders for the three main reflected beams, corresponding to $\lambda = 568$ nm. The period was chosen to 50 pixels and the orientation of the grating was selected to diffract in a direction perpendicular to the multiple reflected beams. This rather large period helps to avoid other secondary effects that affect the diffraction efficiency, such as fringing effect [9].

Figure 8(a) displays the three reflected beams when a uniform screen is addressed to the SLM. The position of the beams (I_1 , I_2 and I_3) is labeled on the picture. The rest of the figures

show images for selected phase values every $\pi/2$ radians. Note that the first reflected beam (I_1) is completely unmodulated. Since it is generated by reflection at the outer surface, it is not modulated by the liquid crystal layer, and no diffraction grating is acting on it. The second beam (I_2) is the main modulated beam, and follows the diffraction efficiency shown in Fig. 7(a). Finally, it is very interesting to compare the diffraction results of the second reflected

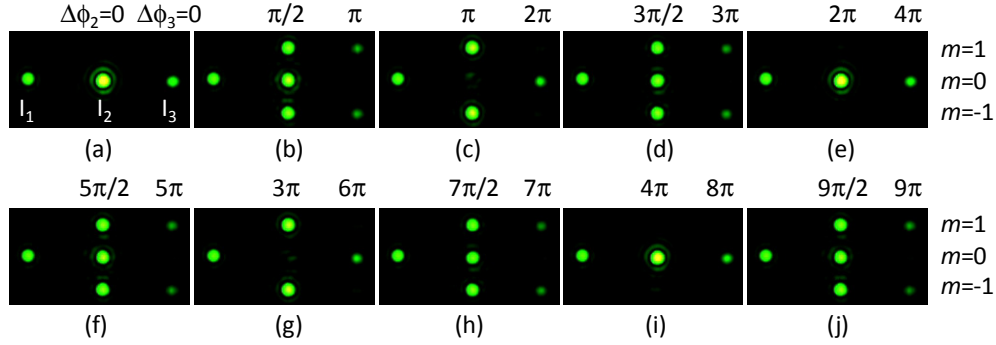


Fig. 8. CCD images of reflected beams (I_1 , I_2 , I_3) with $\lambda = 568$ nm, corresponding to binary gratings with phase difference ranging from 0 to $9\pi/2$ in steps of $\pi/2$. Effective phase modulations $\Delta\phi_2$ for beam I_2 and $\Delta\phi_3 = 2\Delta\phi_2$ for beam I_3 are indicated on top of each image. Diffraction orders $m = 0, \pm 1$ are indicated.

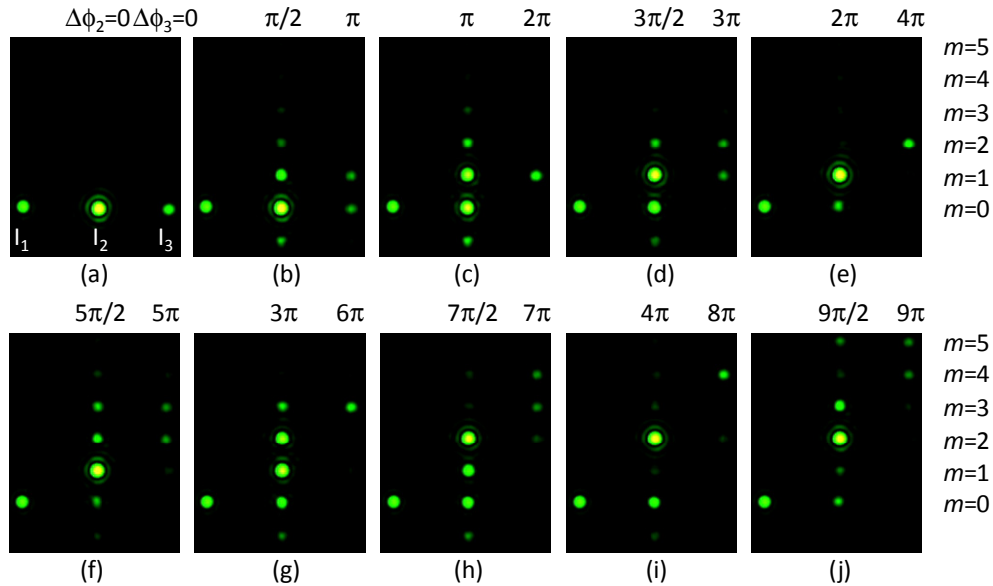


Fig. 9. CCD images of reflected beams (I_1 , I_2 , I_3) with $\lambda = 568$ nm, corresponding to a blazed grating with phase difference ranging from 0 to $9\pi/2$ in steps of $\pi/2$. Effective phase modulations $\Delta\phi_2$ for beam I_2 and $\Delta\phi_3 = 2\Delta\phi_2$ for beam I_3 are indicated on top of each image. Diffraction orders $m = 0, +1, \dots, +5$ are indicated.

beam (I_2) with the third reflected beam (I_3). Since the latter travels twice through the LC layer, the addressed grating acts twice on the beam, and a double effective phase modulation is observed. For instance, in Fig. 8(c), beam I_2 undergoes a complete cancellation of the zero order, indicating a π phase modulation, while beam I_3 shows one single zero diffraction order, indicating a 2π phase modulation. Similarly, in Fig. 8(e) I_2 shows a single zero diffraction order, signaling a 2π phase modulation, as it is the case for I_3 , which is now affected by a 4π

phase binary grating. The same situation is presented in all cases in Fig. 8, and the effective phase modulations $\Delta\phi_2$ and $\Delta\phi_3$ acting on beams I_2 and I_3 are indicated on top of the image, showing that $\Delta\phi_3 = 2\Delta\phi_2$ in all cases.

Figure 9 shows the equivalent results with blazed gratings of the same period. In this case, the zero order does not vanish and other diffracted orders appear, as a direct consequence of the diffraction efficiency degradation provoked by non-perfect phase-only modulation [20]. Nevertheless, the main characteristics of the blazed gratings are observed on the main reflected beam (I_2). When the maximum phase modulation reaches $\Delta\phi_2 = 2\pi$, most of the energy is diffracted onto the first ($m = 1$) diffraction order (Fig. 9(e)), and when it reaches 4π , most of the energy is diffracted onto order $m = 2$ (Fig. 9(i)).

These results also confirm that the third reflected beam (I_3) is affected by a blazed grating with twice the maximum phase modulation than the blazed grating affecting the second reflected beam (I_2), i.e. $\Delta\phi_3 = 2\Delta\phi_2$. For instance Figs. 9(e) and 9(i) show that beam I_3 is mainly diffracted to orders $m = 2$ and $m = 4$, indicating that the effective phase modulation in this reflection is $\Delta\phi_3 = 4\pi$ and $\Delta\phi_3 = 8\pi$ respectively. Similarly, Fig. 9(g) shows that beam I_2 splits mainly to orders $m = 1$ and $m = 2$ indicating a maximum phase shift of $\Delta\phi_2 = 3\pi$ radians in this reflected beam, while beam I_3 diffracts mostly to order $m = 3$, as it corresponds to a maximum phase modulation of $\Delta\phi_3 = 6\pi$. Again, the maximum phase modulation values are indicated on top of each image, and they all agree well with the diffraction results.

7. Conclusions

In summary, we have estimated the complex response of a PAL-LCoS-SLM rendered as a consequence of multiple reflections generated by the intrinsic layer structure of the device. We have shown that maximum phase modulation is achieved at normal incidence whereas increasing the angle of incidence leads to a reduction of the phase modulation range. We have also shown that multiple reflections at the LC layer generate FP interferences that cause a coupled amplitude modulation.

A simplified physical model that accounts for the observed phenomena was developed. By fitting experimental curves for different wavelengths with a reduced set of physical parameters, related to reflection coefficients and LC birefringence, it is possible to quantitatively describe with very good accuracy all the measured modulation curves.

Finally, it was shown that by operating with a large angle of incidence $\theta_0 = 45^\circ$ and using a beam with small diameter, it is possible to spatially isolate different reflected beams and generate interesting effects when addressing phase gratings. A second reflection through the LC layer makes double the effect of the diffraction grating, and doubles the effective maximum phase modulation. This can be a useful effect since it can compensate for the phase modulation reduction that occurs at large angles of incidence.

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