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Ignacio Moreno
Jorge Alberro
Jeffrey A. Davis
Don M. Cottrell
Johnathan B. Cushing



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Ignacio Moreno

Jorge Alberó

Universidad Miguel Hernández
Departamento de Ciencia de Materiales
Óptica y Tecnología Electrónica
03202 Elche, Spain
E-mail: i.moreno@umh.es

Jeffrey A. Davis

Don M. Cottrell

Johnathan B. Cushing

San Diego State University
Physics Department
San Diego, California 92182-1233

Abstract. We used patterned radial polarizers to easily generate and detect radially polarized beams. We then showed how to spatially manipulate the two-dimensional polarization mapping provided by a radial polarizer using different waveplate systems to obtain new two-dimensional polarization states. One system is particularly useful, since it converts the radial polarized beam into the azimuthally polarized beam. The transformed beams were analyzed using linear, circular, and radial polarizers. The Jones matrix formalism was applied for the theoretical analysis. © 2012 Society of Photo-Optical Instrumentation Engineers (SPIE). [DOI: [10.1117/1.OE.51.12.128003](https://doi.org/10.1117/1.OE.51.12.128003)]

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1 Introduction

Radially and azimuthally polarized light beams have been receiving a great deal of attention, especially because they can produce very small focal spots, or generate longitudinal electric field components upon focalization.^{1,2} They can be produced using a variety of techniques, including interferometric systems,^{3–5} optical processing systems,^{6,7} specially designed subwavelength structures,⁸ liquid crystal devices,^{9,10} or inhomogeneous birefringent elements named *q*-plates.^{11,12}

Patterned polarizers are other interesting optical elements that can be used in a variety of applications, including sensors for polarimetric imaging¹³ or phase shifting interferometry¹⁴ employing micropolarizer arrays.¹⁵ Indeed, a patterned azimuthal polarizer was used in combination with a spiral phase plate in Ref. 16, and a patterned radial polarized fabricated with subwavelength gratings was demonstrated in Ref. 17. Although they still present limitations in terms of quantization, due to the limited number of segments, these patterned polarizers represent an extremely simple method to generate radial polarization states.

However, radial and azimuthal polarized beams are only two specific types of vector beams, that is, light beams characterized by a transverse spatial distribution of the state of polarization.¹⁸ Other different polarization spatial distributions can be obtained from the radial or azimuthal polarized beams. Here we use standard half- and quarter-wave plates (HWPs and QWPs, respectively) to spatially transform the polarized beam. There has been little experimental or theoretical work exploring the properties of these other polarization beams using additional optical elements. Our results show new two-dimensional polarization distributions that have not been reported previously to our knowledge.

In this work, we manipulate the two-dimensional polarization pattern provided by a radial polarizer by placing various optical wave plate components after the radial polarizer. One particular case, where a combination of two HWPs creates a polarization rotator, serves to transform the radial polarization into an azimuthal polarization state. The different vector beams generated in this way are then analyzed by

standard linear and circular polarizers, as well as by another radial polarizer.

Note that the generation of radial polarization with patterned polarizers, and its manipulation with waveplates, present advantages with respect to recent developments based on birefringent *q*-plates.¹² These last devices must be precisely tuned to produce a π phase shift, and therefore are useful only for a single wavelength. Changing the wavelength requires retuning the *q*-plate to achieve the correct phase shift. The generation of the radial polarization with radially patterned polarizers offers simplicity and the possibility to be applied over a wide range of wavelengths.

This paper is organized as follows. First, in Sec. 2 we introduce radial and azimuthal polarized beams, along with how they are transmitted by a regular linear polarizer analyzer and also by a radial polarizer analyzer. This analysis is accomplished using the regular Jones matrix formalism. Then, in Sec. 3, we present experimental results that use a patterned radial polarizer as the initial element to generate the radial polarized beam. Next, in Sec. 4 we present the transformation of the radial beam by means of different wave-plate systems. Finally, in Sec. 5 we present the conclusions of the work.

2 Jones Matrix Analysis for Radial and Azimuthal Polarized Beams

The Jones matrix formalism is appropriate to describe the polarization transformations. Next, we apply it to describe the radial and azimuthal polarized beams, and analyze their transmission through polarizer analyzers, including a radial polarizer.

Figure 1 shows a schematic representation of the radial and the azimuthal polarized beams. Let ϕ denote the azimuthal coordinate in the plane perpendicular to the propagation. We select the coordinate $\phi = 0$ corresponding to the horizontal direction. The radial polarized beam [Fig. 1(a)] is a spatially variant polarization beam, where the light is always linearly polarized at every point, but the orientation of the electric field is exactly defined by ϕ . Therefore, it can be described with the following radially polarized Jones vector J_{rad}

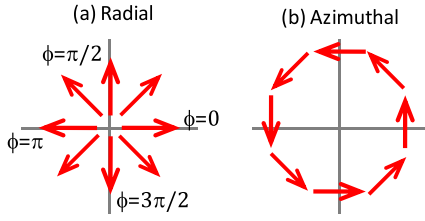


Fig. 1 Schematic representation of the radial and the azimuthal polarized beam.

$$J_{\text{rad}} = \begin{bmatrix} \cos \phi \\ \sin \phi \end{bmatrix}. \quad (1)$$

The azimuthal polarized beam [Fig. 1(b)] is also a spatially variant polarization beam, with the light again linearly polarized at every point, but the orientation of the electric field is rotated by 90 deg with respect to the azimuthal coordinate ϕ . Thus, it can be described by the azimuthally polarized Jones vector J_{azi}

$$J_{\text{azi}} = \begin{bmatrix} \cos(\phi + \frac{\pi}{2}) \\ \sin(\phi + \frac{\pi}{2}) \end{bmatrix} = \begin{bmatrix} -\sin \phi \\ \cos \phi \end{bmatrix}. \quad (2)$$

2.1 Case 1: Analysis of Radial and Azimuthal Polarized Beams with a Linear Polarizer

Let us now consider that these radial and azimuthal polarized light beams traverse a linear polarizer, which acts as an analyzer. If this analyzer transmission axis is oriented at an angle θ , the output Jones vector J_{out} is given, respectively by the equations

$$\begin{aligned} J_{\text{out}}^{\text{rad} \rightarrow \text{linear}} &= \mathbf{P}_{\theta} J_{\text{rad}} \\ &= \begin{bmatrix} \cos^2 \theta & \sin \theta \cos \theta \\ \sin \theta \cos \theta & \sin^2 \theta \end{bmatrix} \begin{bmatrix} \cos \phi \\ \sin \phi \end{bmatrix}, \end{aligned} \quad (3)$$

$$\begin{aligned} J_{\text{out}}^{\text{azi} \rightarrow \text{linear}} &= \mathbf{P}_{\theta} J_{\text{azi}} \\ &= \begin{bmatrix} \cos^2 \theta & \sin \theta \cos \theta \\ \sin \theta \cos \theta & \sin^2 \theta \end{bmatrix} \begin{bmatrix} -\sin \phi \\ \cos \phi \end{bmatrix}, \end{aligned} \quad (4)$$

where

$$\mathbf{P}_{\theta} = \mathbf{R}(-\theta) \mathbf{P}_0 \mathbf{R}(+\theta) = \begin{bmatrix} \cos^2 \theta & \sin \theta \cos \theta \\ \sin \theta \cos \theta & \sin^2 \theta \end{bmatrix} \quad (5)$$

is the Jones matrix of a linear polarizer oriented at an angle θ , and where

$$\mathbf{P}_0 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad (6)$$

is the Jones matrix of a linear polarizer oriented at zero, and

$$\mathbf{R}(\theta) = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \quad (7)$$

is the 2×2 rotation matrix.

Using Eq. (3), we can show that the intensity of the radially polarized beam after traversing the linear polarizer is given by

$$I_{\text{out}}^{\text{rad} \rightarrow \text{linear}} = \cos^2(\phi - \theta). \quad (8)$$

This result shows that the output intensity will be a maximum along the direction of the transmission axis of the linear polarizer.

Similarly, using Eq. (4), the intensity of the azimuthally polarized beam after traversing the linear polarizer is given by

$$I_{\text{out}}^{\text{azi} \rightarrow \text{linear}} = \sin^2(\phi - \theta) = \cos^2\left(\phi - \theta + \frac{\pi}{2}\right). \quad (9)$$

This result shows that, for the case of the azimuthally polarized beam, the output intensity will now be a minimum along the direction of the transmission axis of the linear polarizer.

These two relations show that the intensity of the transmitted beam varies along the azimuthal angle ϕ in a complementary way. For instance, if the linear analyzer is set at $\theta = 0$ deg, Eqs. (8) and (9) become $I_{\text{out}}^{\text{rad} \rightarrow \text{linear}} = \cos^2(\phi)$ and $I_{\text{out}}^{\text{azi} \rightarrow \text{linear}} = \sin^2(\phi)$, respectively.

2.2 Case 2: Analysis of Linear Polarized Beam with a Radial Polarizer

Next we consider the analysis of a linearly polarized beam with a radial polarizer. Radial polarizers are commercially available. In this case, the Jones matrix describing the radial polarizer will be given by an equation similar to Eq. (5), but where the argument is given by the azimuthal angle ϕ ; that is,

$$\mathbf{P}_{\text{rad}} = \begin{bmatrix} \cos^2 \phi & \sin \phi \cos \phi \\ \sin \phi \cos \phi & \sin^2 \phi \end{bmatrix}. \quad (10)$$

One major difference between Eqs. (5) and (10) is that the angle θ in Eq. (5) is a constant value given by the orientation of the linear polarizer, while the angle ϕ in Eq. (10) represents the azimuthal coordinate of the radially polarized beam.

If a homogeneously linearly polarized light beam, with orientation θ , impinges onto the radial polarizer, the output beam can now be described by the following Jones matrix sequence:

$$\begin{aligned} J_{\text{out}}^{\text{linear} \rightarrow \text{rad}} &= \mathbf{P}_{\text{rad}} \begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix} \\ &= \begin{bmatrix} \cos^2 \phi & \sin \phi \cos \phi \\ \sin \phi \cos \phi & \sin^2 \phi \end{bmatrix} \begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix}. \end{aligned} \quad (11)$$

Equation (11) very much resembles Eq. (3). However, there is a major difference. While the output polarization in Eq. (3) is homogeneously linearly polarized at the angle θ , here in Eq. (11) the output polarization is radial; that is, it is identical to the vector in Eq. (1) with an amplitude term $\cos(\phi - \theta)$. Consequently, the intensity of the output beam is identical to Eq. (8), and is given by

$$I_{\text{out}}^{\text{linear} \rightarrow \text{radial}} = \cos^2(\phi - \theta). \quad (12)$$

2.3 Case 3: Analysis of Radial and Azimuthal Polarized Beams with a Radial Polarizer

Using the radial polarizer to analyze radial and azimuthal beams is also interesting. If the radial beam impinges the radial analyzer, the output will be given by the Jones matrix sequence as

$$\begin{aligned} J_{\text{out}}^{\text{rad} \rightarrow \text{rad}} &= \mathbf{P}_{\text{rad}} \begin{bmatrix} \cos \phi \\ \sin \phi \end{bmatrix} \\ &= \begin{bmatrix} \cos^2 \phi & \sin \phi \cos \phi \\ \sin \phi \cos \phi & \sin^2 \phi \end{bmatrix} \begin{bmatrix} \cos \phi \\ \sin \phi \end{bmatrix} \\ &= \begin{bmatrix} \cos \phi \\ \sin \phi \end{bmatrix}. \end{aligned} \quad (13)$$

As expected, the beam remains radially polarized, with uniform maximum intensity transmission $I_{\text{out}}^{\text{radial} \rightarrow \text{radial}} = 1$.

On the contrary, if an azimuthal polarized light beam impinges the radial analyzer, the following result is obtained:

$$\begin{aligned} J_{\text{out}}^{\text{azi} \rightarrow \text{rad}} &= \mathbf{P}_{\text{rad}} \begin{bmatrix} -\sin \phi \\ \cos \phi \end{bmatrix} \\ &= \begin{bmatrix} \cos^2 \phi & \sin \phi \cos \phi \\ \sin \phi \cos \phi & \sin^2 \phi \end{bmatrix} \begin{bmatrix} -\sin \phi \\ \cos \phi \end{bmatrix} \\ &= \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \end{aligned} \quad (14)$$

that is, a null transmission. This shows, in a simple manner, that the radial and the azimuthal beams are two orthogonal axially symmetric light beams.

3 Experimental Results with Radial Polarizers

In our experiments, we used two radial polarizers manufactured by CODIXX.¹⁹ This device consists of angular sectors with linear polarizers, where the polarizer axis is rotated in each angular sector along the corresponding radial direction. Figure 2 shows a scheme of the optical setup. In our experiments, we illuminated the device with an expanded and collimated circularly polarized beam from a He-Ne laser, with $\lambda = 632.8$ nm. A rotating diffuser is added to avoid speckle. Then, the beam transmitted by the radial polarizer is manipulated with wave plates. A lens system is employed to image the input radial polarizer plane onto a charge coupled device (CCD) camera. A polarizer analyzer is placed just in front of the camera.

Figure 3 shows the first set of experimental results. The upper row shows the expected radially polarized output from the CODIXX device. It is important to note that, in an exact radially polarized beam, the electric fields at opposite directions must have the same linear polarization, but with an opposite sense (i.e., with a π phase shift). This is what is indicated as the output from the radial polarizer in the first row in Fig. 4. However, the CODIXX radial polarizer gives the same state (identical polarization and identical phase condition) for the electric fields in the opposite directions. Therefore, the beam emerging from the radial polarizer is not strictly a radially polarized beam. This is visible in the

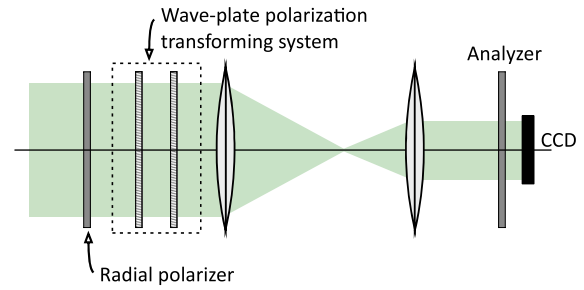


Fig. 2 Schematic representation of the optical setup.

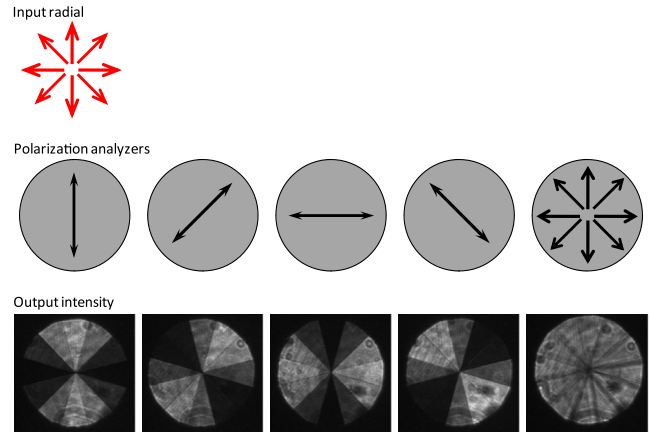


Fig. 3 Experimental transmission when the input beam is radially polarized and the beam is analyzed with a linear or a radial analyzer.

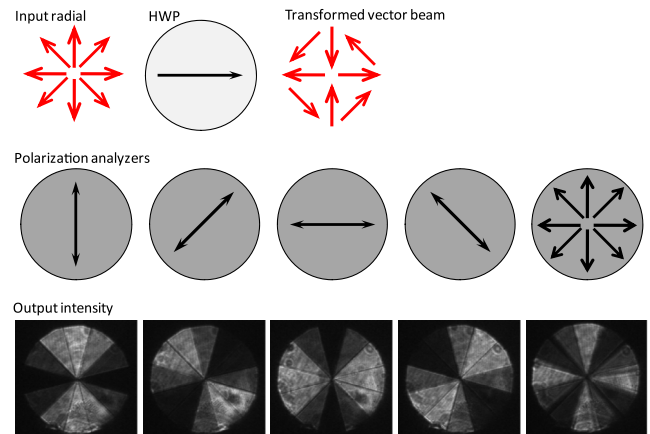


Fig. 4 Experimental transmission when the light emerging from the system composed of the radial polarizer plus a half-wave plate (HWP) oriented at 0 deg is analyzed with a linear or a radial analyzer.

absence of the dark point at the center of the beam, corresponding to the created singularity. A π -phase shift in one half of the polarizer with respect to the other would be required to achieve this perfect situation. This is, for instance, how truly radial and azimuthal polarized beams can be generated using liquid crystal radial polarization converters, which include a half-plane variable retarder cell that is adjusted to compensate this π phase shift.²⁰ In spite of this effect, the intensity transmission obtained through different analyzers (which is the aim of this work) will be the same regardless of this relative π phase shift.

The middle row in Fig. 3 shows the various polarizer analyzers, including a linear analyzer, which is oriented at different angles of $\theta = 0, \pm 45$ deg and 90 deg, and a second CODIXX radial polarizer. The bottom row shows the experimental output intensity, captured by the CCD camera. As expected, these light intensity patterns show that the beam is radially polarized. The transmitted light intensity is strongest along the axis of the analyzer polarizer, in agreement with Eq. (8). Unfortunately, we see that the device has a small number of sectors, which limits the continuity of the radially polarized beam. The result presented in the bottom row, right image, shows the output when the input beam passes through a second radial polarizer device, also manufactured by the same company. As expected, the radially polarized beam is fully transmitted by the radial polarizer, in agreement with Eq. (13).

Next we show that the radial polarized beam can be manipulated with wave plates. In fact, we show how to generate, in a simple manner, the azimuthal polarized beam out of the radial polarized beam with two HWPs.

4 Polarization Manipulation of the Radial Polarized Beam

We start by analyzing the transformation induced by a HWP. A HWP will rotate the plane of polarization by twice the angle between the incident polarization direction and the wave plate principal axis. The Jones matrix corresponding to the HWP with its principal axis oriented at an angle θ with respect to the x reference axis can be written as

$$\mathbf{HWP}_\theta = \mathbf{R}(-\theta) \cdot \mathbf{HWP}_0 \cdot \mathbf{R}(\theta) \\ = \begin{bmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{bmatrix}, \quad (15)$$

where

$$\mathbf{HWP}_0 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \text{and} \quad \mathbf{HWP}_{45^\circ} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad (16)$$

are the HWP matrices corresponding to $\theta = 0$ and $\theta = 45$ deg, respectively.

The radially polarized beam from the CODIXX device that traverses the HWP oriented at $\theta = 0$ is given by the next Jones matrix sequence:

$$\mathbf{J}_{\text{out}} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \cos \phi \\ \sin \phi \end{bmatrix} = \begin{bmatrix} \cos \phi \\ -\sin \phi \end{bmatrix}. \quad (17)$$

This space variant polarized vector beam is shown in the top right of Fig. 4 and represents a new two-dimensional polarization state. We see that the horizontal regions are unaffected. However, the polarization directions of the vertical regions are reversed. In addition, the polarization axes at $\phi = 45$ deg are rotated by 90 deg. When this beam traverses a linear analyzer with orientation θ , following identical Jones calculations as in Sec. 2, the transmitted intensity is now given by $I_{\text{out}}^{\text{linear}} = \cos^2(\phi + \theta)$. When the radial polarizer is used as the analyzer, the output intensity is $I_{\text{out}}^{\text{radial}} = \cos^2(2\phi)$. This last result shows that the intensity transmission oscillates with twice the azimuthal coordinate.

The experimental results are shown in Fig. 4. The second row shows the orientations of both the linear polarizer and

the radial analyzer. The bottom row shows the corresponding transmission intensities captured by the CCD camera, where the output is polarized as expected. The light intensity is brightest along the directions where the polarization state in the middle column is aligned with the polarization axis of the analyzer polarizer. The pattern when the radial analyzer is employed has four bright and four dark regions along the azimuthal direction, showing a Maltese cross-type pattern, which corresponds to the 2ϕ oscillation described above.

In Fig. 5, the HWP is rotated by 45 deg. As before, the top row shows the incident polarization state, the orientation of the HWP, and the expected output polarization map of the transformed vector beam. This output polarization can be calculated as

$$\mathbf{J}_{\text{out}} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \cos \phi \\ \sin \phi \end{bmatrix} = \begin{bmatrix} \sin \phi \\ \cos \phi \end{bmatrix}. \quad (18)$$

Again, the beam represents a new two-dimensional polarization state that has not been reported before to our knowledge. We see that the polarization in the horizontal and vertical regions are now rotated by 90 deg. On the contrary, the polarization direction along the axis of the HWP is unaffected, while the polarization directions perpendicular to the axis of the HWP are rotated by 180 deg. The second row shows, as previously, the orientations of the linear polarizer analyzer as well as the radial polarizer analyzer. The intensity in each case is now $I_{\text{out}}^{\text{linear}} = \sin^2(\phi + \theta)$, and $I_{\text{out}}^{\text{radial}} = \sin^2(2\phi)$ (i.e., complementary with respect to those in Fig. 4). The bottom row shows the experimental results, verifying that the output is polarized as expected. The light intensity is brightest along the directions where the polarization state is aligned with the polarization axis of the linear polarizer analyzers. Now, when analyzed by the radial analyzer, the same Maltese cross-type pattern as in Fig. 3 is obtained, but rotated by 45 deg.

In Fig. 6, we combine the two HWPs. The optical axis of the first HWP is oriented at 45 deg while the optical axis of the second HWP is oriented in the horizontal direction. Note that the Jones matrix of the combination of two HWPs is given by:

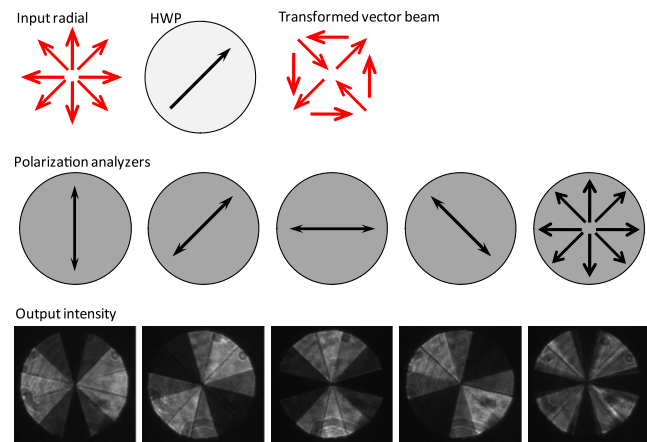


Fig. 5 Experimental transmission when the light emerging from the system composed of the radial polarizer plus a HWP oriented at 45 deg is analyzed with a linear or a radial analyzer.

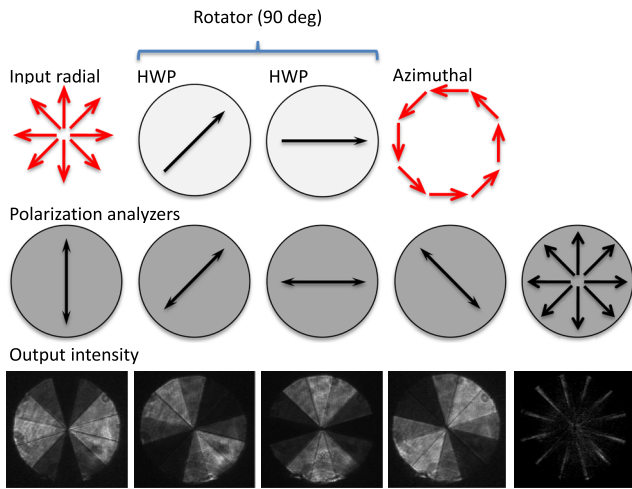


Fig. 6 Experimental transmission when the light emerging from the system composed of the radial polarizer plus a 90 deg polarization rotator is analyzed with a linear or a radial analyzer.

$$\begin{aligned} \text{HWP}_0 \cdot \text{HWP}_\theta &= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{bmatrix} \\ &= \begin{bmatrix} \cos(2\theta) & \sin(2\theta) \\ -\sin(2\theta) & \cos(2\theta) \end{bmatrix}. \end{aligned} \quad (19)$$

This result shows that this combination acts as a pure polarization rotator, where the rotation angle is twice the relative angle between the principal axes of the wave plates. In this case, since $\theta = 45^\circ$ we obtain a 90° deg polarization rotator. When the radially polarized beam passes through this polarization rotator, the result is the azimuthally polarized beam of Eq. (2). As before, the first row in Fig. 6 shows the incident polarization state, the orientations of the HWPs, and the expected azimuthally polarized light. The experimental images in the bottom row show that this expected output is obtained. The light intensity is brightest along the directions where the polarization state is aligned with the transmission axis of the linear polarizer analyzer, and the results are in agreement with Eq. (9). The right side of the bottom row shows the case where the analyzer is the second radial polarizer. Since the azimuthally and radially polarized beams are orthogonal, there is no output [Eq. (14)]. Only the edges of the angular sectors are visible. Note that the orthogonality of the radially and azimuthally polarized beams makes them interesting as another possible basis for polarization decomposition, and a new polar-Jones vector representation based on them has been proposed recently.²¹

Note that the radial polarizer– 90° deg polarization rotator combination can be viewed in two ways. When the rotator is placed behind the radial polarizer, the system acts as an azimuthally polarized beam generator. However, when the rotator is placed before the radial polarizer analyzer, the system acts as an azimuthal analyzer (which completely absorbs the incoming radially polarized beam).

Figure 7 shows an interesting variation on the setup in Fig. 6. Here we combine again two HWPs. However, the optical axis of the second HWP is now oriented at 22.5° . The combination of both HWPs acts as a 45° deg polarization rotator. The output transformed vector beam is then a spiral-polarized beam as shown in the top right of Fig. 7. This type

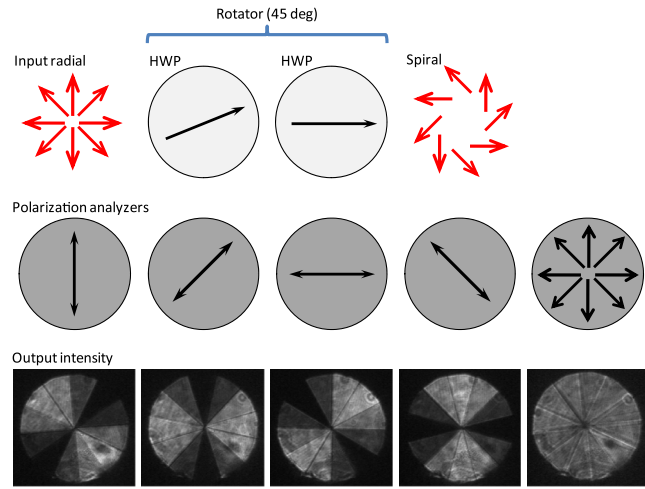


Fig. 7 Experimental transmission when the light emerging from the system composed of the radial polarizer plus a 45° deg polarization rotator is analyzed with a linear or a radial analyzer.

of beam was reported in Ref. 5 using a much more complex optical system. Note that the Jones vector describing this spiral polarized beam is given by

$$\begin{aligned} J_{\text{out}} &= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \cos \phi \\ \sin \phi \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} \cos(\phi) - \sin(\phi) \\ \sin(\phi) + \cos(\phi) \end{bmatrix} \\ &= \frac{1}{\sqrt{2}} \begin{bmatrix} \cos \phi \\ \sin \phi \end{bmatrix} + \frac{1}{\sqrt{2}} \begin{bmatrix} -\sin \phi \\ \cos \phi \end{bmatrix}. \end{aligned} \quad (20)$$

This equation shows that this spiral polarization can be regarded as a linear combination, with equal weight, of a radially polarized beam and an azimuthally polarized beam. When this beam is transmitted by a linear analyzer with orientation θ , the transmission is given by $I_{\text{out}}^{\text{linear}} = \cos^2[\phi + (\pi/4) - \theta]$, which is equivalent, except for a $\pi/4$ rotation, to that obtained when the light impinging on the analyzer is the radial beam [Eq. (8)]. On the contrary, when the spiral polarized beam is analyzed with the radial polarizer, the transmission is uniform at $1/2$ value, corresponding to the intensity of the radial component in the decomposition in Eq. (20).

Again, the first row in Fig. 7 shows the incident polarization state, the orientations of the HWPs and the expected output polarization map of the transformed vector beam. The bottom row shows that the output is polarized as expected. The light intensity is brightest along the directions where the polarization is aligned with the transmission axis of the analyzer. The right image in the bottom row shows the case where the analyzer is the second radial polarizer and we see that half of the light is uniformly transmitted.

Finally, Figs. 8 and 9 show some additional interesting results, by transforming the radially polarized beam with a QWP. Figure 8 shows the case where the QWP is along the horizontal axis. The Jones matrix transformation is given by:

$$J_{\text{out}} = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \begin{bmatrix} \cos \phi \\ \sin \phi \end{bmatrix} = \begin{bmatrix} \cos \phi \\ i \sin \phi \end{bmatrix}. \quad (21)$$

This shows that the output is elliptically polarized and centered on the xy coordinate axes. Now the beam is left and right circularly polarized at all four quadrants (when $\phi = 45^\circ, 135^\circ, 225^\circ$, and 315°). Again, this beam represents a new two-dimensional polarization state.

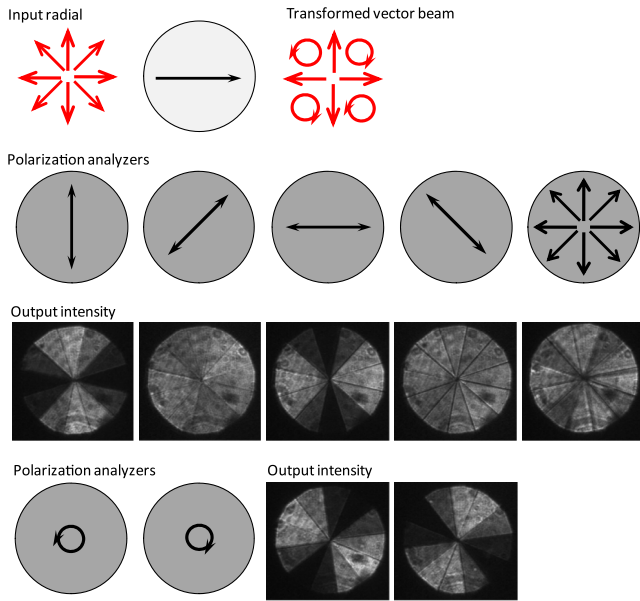


Fig. 8 Experimental transmission when the light emerging from the system composed of the radial polarizer plus a QWP oriented at 0 deg is analyzed with linear, radial, and circular analyzers.

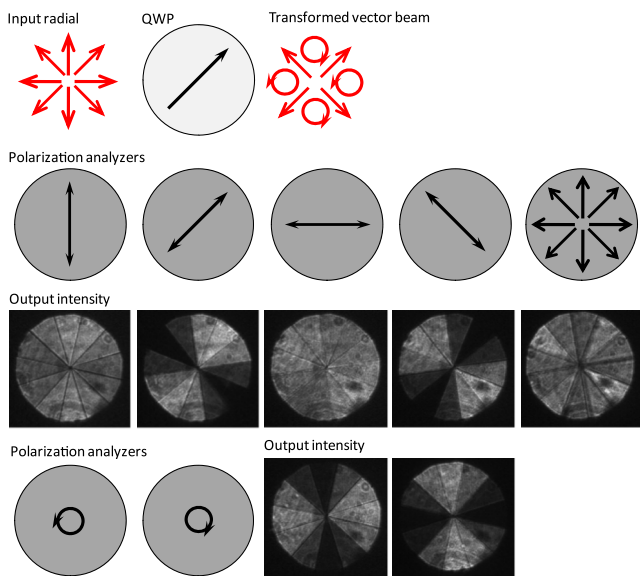


Fig. 9 Experimental transmission when the light emerging from the system composed of the radial polarizer plus a QWP oriented at 45 deg is analyzed with linear, radial, and circular analyzers.

When this beam traverses the linear polarizer with orientation θ , the transmitted intensity becomes $I_{\text{out}}^{\text{linear}} = \cos^2(\theta)\cos^2(\phi) + \sin^2(\theta)\sin^2(\phi)$. Note that when $\theta = \pm 45$ deg, the transmission is uniform with a value $1/2$. This is proven with the corresponding experimental result shown in the figure. When the radial polarizer is used as analyzer, the transmitted intensity is given by $I_{\text{out}}^{\text{radial}} = \cos^4(\phi) + \sin^4(\phi)$. This reproduces the same Maltese cross as in Fig. 4, but with less contrast since complete absorption is never produced and dark areas are not produced. Finally, we proved that the beam at the four quadrants is circularly polarized. The results by the bottom row in Fig. 8 are obtained when we use a left or right circular polarizer

analyzer and prove the circular polarization states along the four quadrant directions.

Figure 9 shows equivalent results when we place the QWP with its optic axis at 45 deg. Now the input polarization state is not changed if it is parallel or perpendicular to the wave-plate optical axis (i.e., at the four quadrants). However, when the input polarization state is at ± 45 deg to the optical axis, then the polarization state is either left or right circularly polarized. This situation happens now along the horizontal and vertical axes. Once again, this beam represents a new two-dimensional polarization state that has not been reported before to our knowledge.

As in the previous case, when analyzed with the linear polarization analyzer, the beam is transmitted uniformly when the analyzer axis forms an angle of ± 45 deg with the principal axis of the QWP (i.e., when the linear analyzer is at horizontal or vertical in this case). When the radial analyzer is employed, again the same Maltese cross as in Fig. 5 appears, but with less contrast. Finally, in the bottom row, the use of the circular polarization analyzers prove the circular polarization states along the horizontal and vertical directions.

5 Conclusions

In summary, we used two commercial patterned radial polarizers to generate and detect a radially polarized light beam. Although they are still very expensive in comparison to regular linear or circular homogeneous polarizers, we show that such patterned polarizers might become a very useful element for polarization engineering and polarization analysis. They can generate, simply by transmission, a pseudo-radially polarized beam (note this is not a perfect radially polarized beam because of a π phase shift difference in opposite segments). This simple transformation, in comparison with other alternative methods in the literature, make them valuable optical elements that might stimulate the use of radially polarized beams.

Then, we showed some interesting ways to manipulate the two-dimensional polarization map produced by a radial polarizer, by means of different wave plate combinations. One of them is of particular interest, since a polarization rotator composed of two HWPs generates the azimuthal polarized beam out of the radially polarized beam. Therefore, such a polarization rotator converts the radial polarizer into an azimuthal polarizer. We provided a complete Jones matrix analysis of the transformations induced by the wave plates, as well as for the transmission through the different final polarizer analyzers. Experimental results agree with theory in all cases.

In addition, by using various combinations of different waveplates, we can generate a number of new two-dimensional polarization states that have not been reported before to our knowledge. These kinds of manipulations of the two-dimensional polarization map can be made programmable by inserting either one or two liquid crystal wave plates after the radial polarizer.²² In this promising approach, the optical axis of one wave plate would be oriented in the horizontal direction while the optical axis of the second programmable wave plate would be oriented at 45 deg. By varying the phase shift introduced by these two wave plates, we can easily recreate all of the results shown here. The case where the 90 deg polarization rotator is employed to convert the radial polarization into the azimuthal polarization can be of particular interest,

since the use of a programmable wave plate system will permit rapid switching between these two types of beam.

Acknowledgments

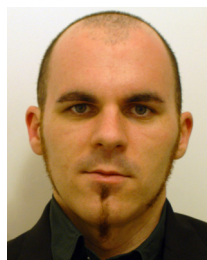
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Ignacio Moreno completed his BS degree in physics and PhD in physics at the Autonomous University of Barcelona (Spain) in 1992 and 1996, respectively. His research interests include liquid crystal displays and its applications in diffractive and polarization optics, being coauthor of 100 publications in peer-reviewed journals. He is a senior member of SPIE and the Optical Society of America. He is currently the editor of the journal *Óptica Pura y Aplicada* (OPA), edited by the Spanish Optical Society.



Jorge Albero received the Telecommunication Engineering degree from Universidad Miguel Hernández de Elche, Spain, in 2006. He received his PhD degree at the Institute FEMTO-ST from the Université de Franche-Comté Besançon (France) in 2010. He was preengineering research student at the Wrocław University of Technology for six months. In 2011 to 2012, he was researcher at Universidad Miguel Hernández de Elche, under a postdoctoral scholarship from the Spanish Ministry of Education, and he is currently researcher at Institute FEMTO-ST (France). His research interests concern microoptics fabrication for MOEMS and integrated microscopy with spatial light modulators.



Jeffrey A. Davis received his BS degree in physics from Rensselaer Polytechnic Institute and his PhD from Cornell University. He is currently a professor of physics at San Diego State University, where he leads the electro-optics program. His research interests include optical pattern recognition, spatial light modulators, and programmable diffractive optical elements. He is coauthor of more than 150 publications in peer-reviewed journals. He is a fellow of SPIE and the Optical Society of America.



Don M. Cottrell received his BD and PhD degrees from the University of Washington. He is Emeritus professor of physics at San Diego State University. His research interests include the computer construction of synthetic holograms. He also conducts theoretical studies in general relativity and particle physics involving differential geometry and supersymmetry.



Johnathan B. Cushing is a graduating junior at San Diego State University who is currently doing research on light polarization states and the altering of beams through optical systems. After serving six years in the United States Navy, he is working toward his PhD in physics in order to continue serving his country as a researcher or professor.